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MULTI-MODEL METHODS AND PARAMETERS ESTIMATION APPROACHES ON NON-LINEAR DYNAMIC SYSTEM IDENTIFICATION

Abstract. In this article 3 non-linear system identification methods are investigated. Advantages and drawbacks are studied. Test system identification process simulation with proposed methods are conducted.
Keywords: non-linear dynamic system identification, multi-model identification methods, simulation.

Introduction

Identification of the complex non-linear dynamic systems, in particular, systems with chaotic dynamic is a challenging task. Numerous methods can be applied in this area, but, obviously, the is no "silver bullet" due to unlimited set of non-linear systems. One of the most famous adaptive-searching identification methods [1,3], in case of adequate criterion is provided, can be successfully used for this purpose. But, as shown in [2], this methods have some drawbacks. First of all, this methods spend too much time to locate criterion extremum. Other essential drawback – measurement near one point decreases probability of identification in case of multi-extremum criterion shape. Usage of five fixed model approach, as shown in [2], can significantly increase identification speed. Backside result of this method – loss of accuracy, especially far for fixed model points.

So, the actual problem is to create methods, which combine positive features of adaptive-searching and fixed multi-model approaches.

Task definition

To receive identification simulation results, which is independent of particular dynamic system properties, the model of identification error (or the identification criterion) is required. One of the reasons of accuracy loss assumed to be criterion non-symmetrical form. So, the model must have a uniformly controlled part, which describes such phenomena. In this paper one of the simple representation will be used:

$$q_{mi}\left(p_o, p_{mi}\right) = a_q\left(p_o - p_{mi}\right) + a_{qm}\left|p_o - p_{mi}\right|, \tag{1}$$

where p – parameter, q – criterion, a_q – sensitivity coefficient, a_{qm} – non-linearity coefficient. Index "*o*" belongs to object under identification, and index "*mi*" – to model number *i*.

Function of quality F represented by this way:

$$F(q_{mi}) = \exp\left(-\frac{q_{mi}^2}{q_\gamma^2}\right), \tag{2}$$

where q_γ – sensitivity scale. This definition differs from usual, where numerator have part like $\left(q_o - q_{mi}\right)^2$, but allows us to uniformly control asymmetrical properties. Usage of definitions (1) and (2) hides from consideration real system dynamic, object and model outputs $x_o\left(t\right)$, $x_{mi}\left(t\right)$, and describes all properties to 2 coefficients: a_{q1} and a_{qm}. This simplification give chance to determine common properties of identification system itself. We assume, that dynamic properties on identification system is much "slower", then non-linear system under consideration.

In paper [2], 5 models with fixed parameters was used, and final parameter value was defined in form, close to fuzzy logic "centre of gravity" approach:

$$p_{ge} = \frac{\sum_{i=0}^{n-1} F_{mi} p_{mi}}{\sum_{i=0}^{n-1} F_{mi}}, \tag{3}$$

where n – number of models, and index "*ge*" denotes "global extremum". All definitions is converted to used in this paper, not to disturb attention by different definitions.

One clearly visible drawbacks was non-uniform usage of models at bounds, namely number 0 and number (n–1). As a countermeasure of we add 2 fake fixed "out of band" models, designated by indexes "*ll*" and "*rr*", and with a fixed criterion values: $q_{ll} = q_{rr} = 0$. As initial parameter values of real models distributed uniformly on working range, parameter values of fake models defined as: $p_{ll} = p_{m0} - \left(p_{m1} - p_{m0}\right)$ and $p_{rr} = p_{m(n-1)} + \left(p_{m1} - p_{m0}\right)$.

Calculation of p_{ge} is quite simple, but in case of multi-extremum criterion shape, or large q_γ value, the influence of the models, which is far from real extremum, may be significant. Define value of p_{le} to drop such influence. Let i_m – index of model with maximum *F*. Then

$$p_{le} = \frac{F_{i_m-1}p_{i_m-1} + F_{i_m}p_{i_m} + F_{i_m+1}p_{i_m+1}}{F_{i_m-1} + F_{i_m} + F_{i_m+1}} \quad . \tag{4}$$

If $i_m - 1$ or $i_m + 1$ is out of bounds, substitute corresponding values of one of fake models.

Another way to determine extremum point in the range of 3 adjutant models is to approximate *F*(*p*) by parabola. Let index "c" designated "current" value of *i*, and "l" means (c−1), "r" – (c+1). To simplify calculations, move the coordinate origin, so $\tilde{p}_c = (p_c - \mathrm{p}_c) = 0$, $\tilde{p}_l = (p_l - p_c)$, $\tilde{p}_r = (p_r - p_c)$, $\tilde{F}_c = F(p_c) - F(p_c) = 0$, $\tilde{F}_l = F(p_l) - F(p_c)$, $\tilde{F}_r = F(p_r) - F(p_c)$. Thus:

$$\begin{cases} a_2\tilde{p}_l^2 + a_1\tilde{p}_l = \tilde{F}_l \\ a_2\tilde{p}_r^2 + a_1\tilde{p}_r = \tilde{F}_r \end{cases}, \tag{5}$$

$$a_1 = \frac{\tilde{F}_r\tilde{p}_l^2 - \tilde{F}_l\tilde{p}_r^2}{\tilde{p}_l^2\tilde{p}_r + \tilde{p}_l\tilde{p}_r^2},$$

$$a_2 = -\frac{\tilde{F}_r\tilde{p}_l - \tilde{F}_l\tilde{p}_r}{\tilde{p}_l^2\tilde{p}_r + \tilde{p}_l\tilde{p}_r^2},$$

$$\tilde{p}_e = -\frac{a_1}{2a_2},\ p_e = p_c - \frac{a_1}{2a_2}. \tag{6}$$

To prevent displacement of p_e out of band, defined by current 3 models, limit p_e to $(p_l; p_r)$. For practical reason, limitation may be stronger.

The value of p_e where $c = i_m$ define as p_{ee}. While simulation of identification process, we will determine p_{ge}, p_{le} and p_{ee}, along with corresponding identification errors:

$$e_{ge} = p_o - p_{ge}, \quad e_{le} = p_o - p_{le}, \quad e_{ee} = p_o - p_{ee}\,. \tag{7}$$

The quality of identification will be estimate as standard deviation of identification errors on all simulation time *T*.

Identification process simulation

The identification process simulation was conducted by developed simulation program "qontrol". As mentioned before, instead of real object identification, was used criterion approximation, given by (1). To check ability to identify non-steady systems, object parameter p_o was given by such equation:

$$p_o(t) = U_{in} \sin(\omega_{in} t) + C_0, \tag{8}$$

where coefficients was set to values, which allows to test different modes: $U_{in} = 40$; $\omega_{in} = 1.1$; $C_0 = 45$. Working parameter range was given as $(0,100)$. Initial values of models parameter: $p_{ll} = -10$; $p_{m0,0} = 10$;

$p_{m1,0} = 30$; $p_{m2,0} = 50$; $p_{m3,0} = 70$; $p_{m4,0} = 90$; $p_{rr} = 110$.

The values of other parameters: $a_q = 5$; $a_{qm} = 0.5$ (small) or $a_{qm} = 2.0$ (large asymmetry). The value of q_γ was changed in range [2;140], standard deviations of identification errors was measured, and plots are provided for better value of q_γ.

In fig. 1 represented simulation results for identification system with n=5 and $q_\gamma = 60$. The identification error analyses shows, that in this environment there is no visible difference between p_{eg} and p_{el}. As strange as it seen, the values of p_{ee} shows the worst results. And obviously, error is minimal near one of model parameters, and essentially increased if $p_o \notin [p_{m0,0}; p_{m4,0}]$.

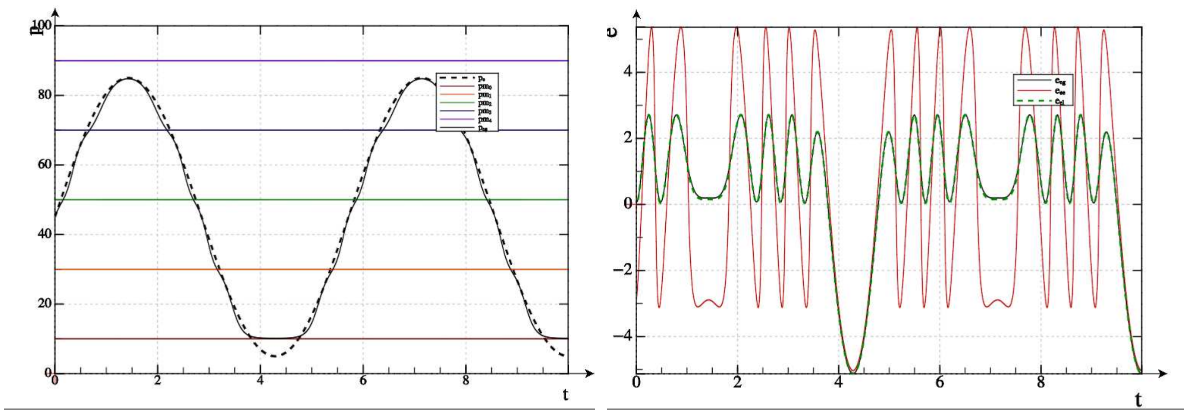

Figure 1 – Identification with fixed 5 models and 2 fake models

In fig. 2 represented dependencies of $\bar{e}_{eg}$, $\bar{e}_{el}$ and $\bar{e}_{ee}$ from q_γ at different values of a_{qm}.

This results shows, that for every method of p estimation, an optimal q_γ value exists, large values of a_{qm} leads to large errors, and optimal q_γ value for $\bar{e}_{ee}$ is essentially large, then for $\bar{e}_{eg}$, $\bar{e}_{el}$.

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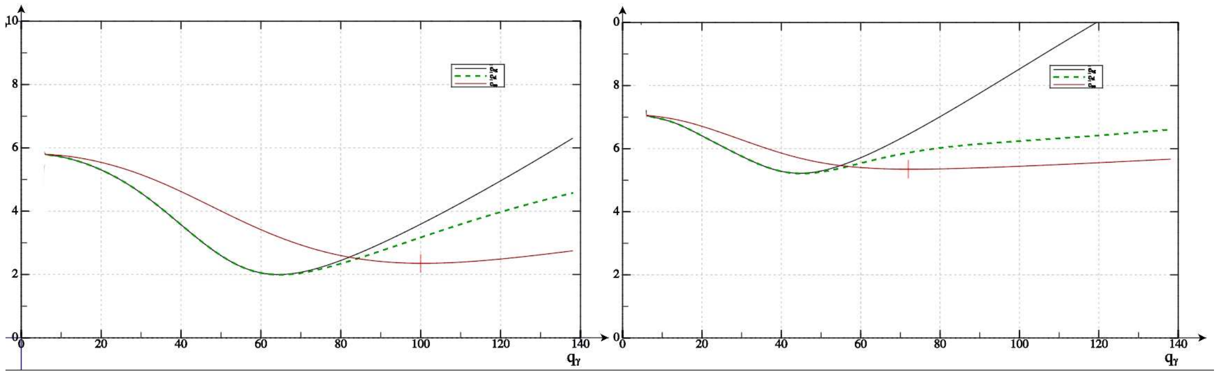

Figure 2 – Error dependencies from q_γ while identification with fixed 5 models and 2 fake models, $a_{qm} = 0.5$ (left), and $a_{qm} = 2.0$ (right plot)

Moving bang-limited models identification system

To achieve better results, lets allow model to change own parameter. The new parameter value p_e is given by (6). I.e. every model (except fake) takes in account not only own parameter and quality function, but the same of nearest neighbours. But if we allow for all models to move freely, all models will fall in one small area, and not react to next parameter movement. To prevent this, we assign a non-intersected band for every model, which limits parameter. The simulation results are shown in fig. 3 and 4.

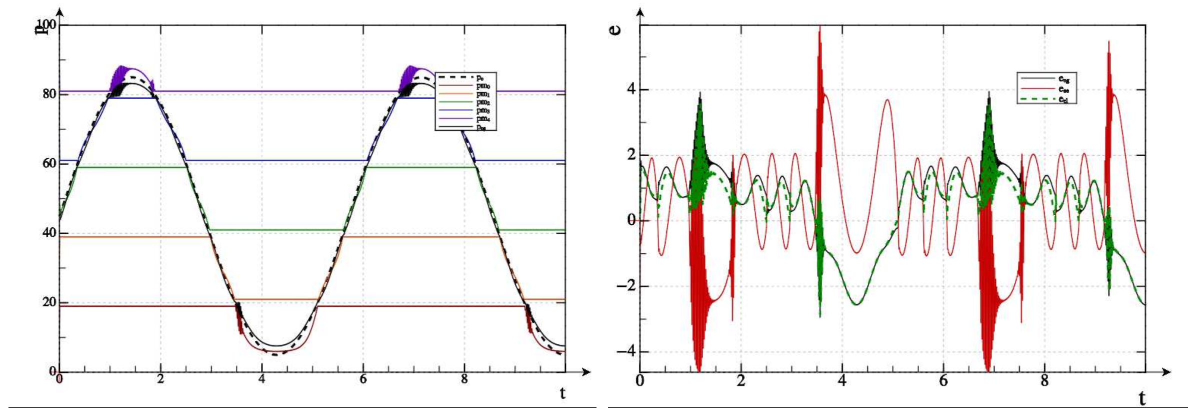

Figure 3 – Identification with 5 band-bounded models and 2 fake models

The result shows, that, in general, this method leads to less errors with the same conditions. Moreover, due to moving parameters the are no “dead zones” near working range boundaries. One model is follows the object parameter in own band, and provides better identification results.

The error shapes shows one drawback for this method: near the bands boundaries the are fast switching oscillation. This is due to the fact, that identification process is dynamical too, and we should not neglect its dynamic.

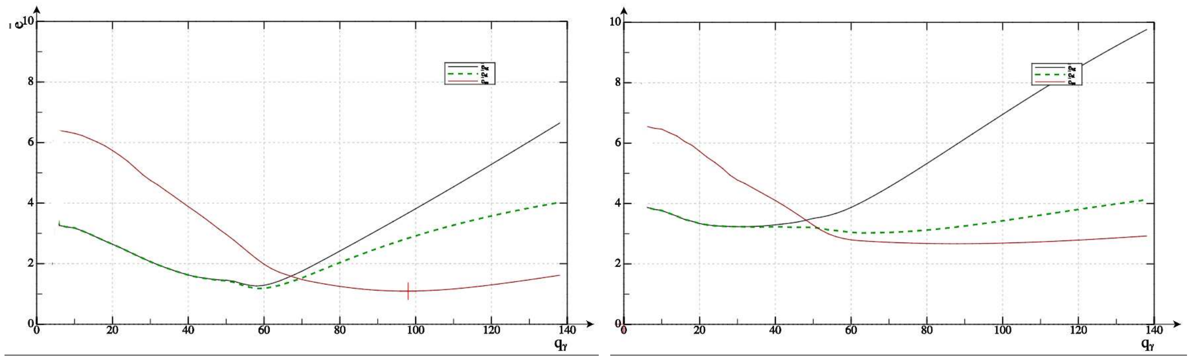

Figure 4 – Error dependencies from q_γ while identification with 5 band-bounded models and 2 fake models, $a_{qm} = 0.5$ (left), and $a_{qm} = 2.0$ (right plot)

Method with forces and viscous

To achieve better results, we can treat every real model as physical body under influence of different forces. We consider given forces:

1. $f_c = -k_c\left(p_c - p_{c,0}\right)$ – return force to model starting point;
2. $f_n = k_n\left(p_r - 2p_c + p_l\right)$ – force due fo displacement from neighbour model;
3. $f_e = -k_e\left(p_c - p_e\right)$ – force caused by displacement from locally estimated extremum point (6).

Total force $f_t = f_c + f_n + f_e$ may lead to model movement by different ways. In this paper the viscous approach is used:

$$\frac{dp}{dt} = \upsilon_f f_t\left(p\right) \tag{9}$$

The simulation results are shown in fig. 5 and 6. Used parameters: $k_c = 1$, $k_n = 1$, $k_e = 5$, $\upsilon_f = 2$.

The simulation results shows, then this method shows best results among approaches under consideration. At least 2 models follows the object parameter, that gives good accuracy. Other models moved to extremum too, but continues to watch working range.

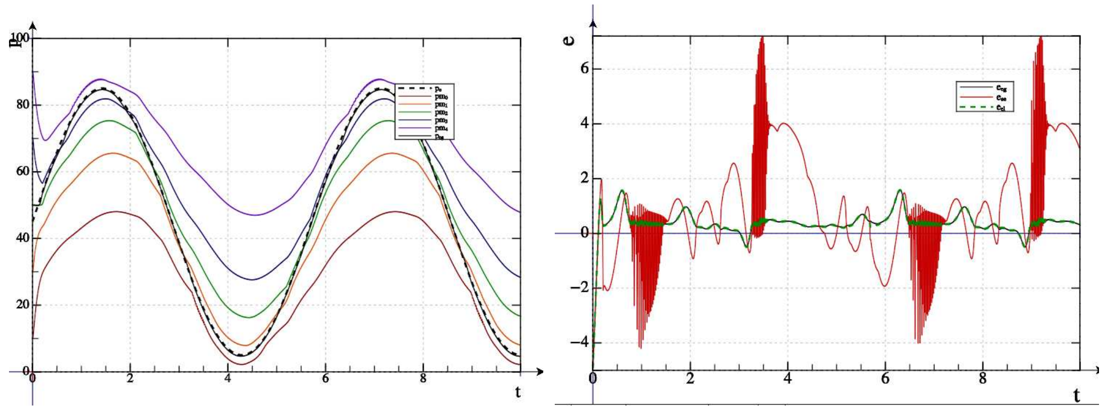

Figure 5 – Identification with 5 models with forces and 2 fake models

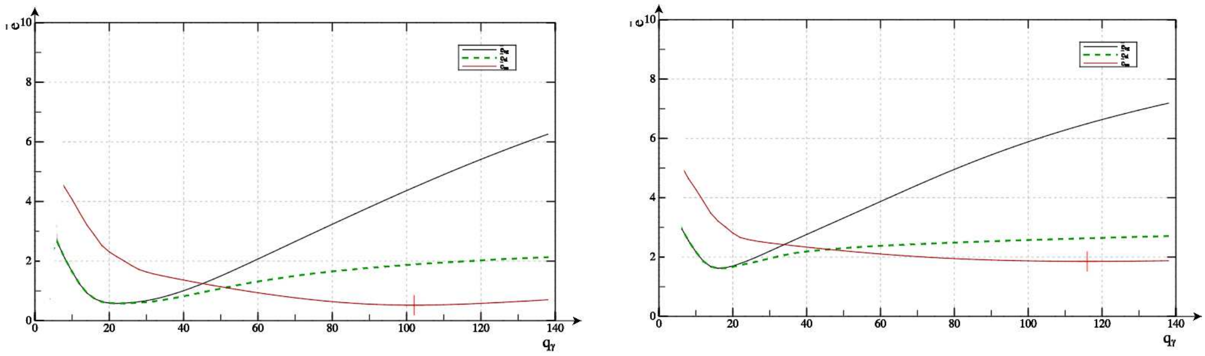

Figure 6 – Error dependencies from q_γ while identification with 5 models with forces and 2 fake models, $a_{qm} = 0.5$ (left), and $a_{qm} = 2.0$ (right plot)

Conclusions

Results of identification process simulation allow us to make some conclusions:

- identification with fixed models, inspite of simplsity and speed, can not give good accuracy and full range covering;
- system with band - limied parameters gives better results;
- system with models, which parameters displacement is described as body movement under forces gives best result.
- in conditions under consideration, there is no valuable difference between p_{eg} and p_{el}.

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