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DESIGN OF THE AUTOMATED COMPLEX FOR MEASUREMENT OF THE COMPLEX REFLECTION COEFFICIENT IN THE MICROWAVE RANGE BASED ON THE PRINCIPLES OF HOLOGRAPHY

Abstract. For 6-port vector analyzer based on the E-plane waveguide cross-connection of rectangular waveguides, the benefits of determining the values of the complex reflection coefficient with the application of the principles of holography with three reference signals were approved. The value of the condition number of the matrix system of linear algebraic equations formed according to the methods processing considered was used as a criterion for the selection of the waveguides connection angle. It is shown that the allowable angle is 60 degrees.

Keywords: complex reflection coefficient, holography, regularization, waveguide cross.

Introduction. Microwave methods are widely used to solve many production problems in metallurgy, mechanical engineering, particularly in non-destructive testing [1]. In fact, the use of these methods is based on the measurement of the frequency dependence of the complex reflection coefficient of the object of interest. An essential part of modern means of measuring the complex coefficient of reflection such as vector reflectometer is based on the idea of a 6-port meter [2]. The traditional scheme requires the use of a significant number of standard loads and the use of well-matched microwave elements, which are extremely expensive. All this leads to the impossibility of using this approach directly in an industry. Scheme of 6-port meter is being considered in [3] is based on E-plane waveguide cross-junction of simple construction which provides possibility of scattering matrix elements calculation by rigorous means without the number of calibration procedures thus the cost of the instrumentation is comparable to the value of the scalar reflectometer. In this case, the direct angle of connection waveguides in the cross-junction was under consideration. The traditional approach to determining the value of complex reflection coefficient under interest is a simultaneous solution of the three quadratic equations. In fact, the concept of the 6-port meter implements a holographic recording with three reference

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signals [4], there may be formed a special system of linear equations. This approach makes it possible to analyze the accuracy characteristics for different schemes based on the analysis of the condition number of the corresponding matrix with dimension of 2×2. In [5] the cross-shaped connection of waveguides at an arbitrary angle is considered.

Problem Formulation. The purpose of this paper is to analyze the construction of 6-port meter on the basis of the E-plane cross-junction of rectangular waveguides with arbitrary angle of connection using the behavior of the condition number of the corresponding matrix realizing calculations according to the principle of holography with three reference signals.

Main Part. The cross-junction divides the input power in four approximately equal parts. It serves simultaneously for summation of a signal under test with three reference signals obtained by dividing the input signal. Peculiarity of use of the cross-junction is an opportunity to evaluate scattering matrix elements with use of rigorous electromagnetic methods excluding calibration procedure. The cross-junction has rather smooth frequency characteristics in whole operating band of a rectangular waveguide. As result, only the coefficients of detector characteristics in measuring ports should be determined by calibration. It is clear that the matched load is most suitable for calibration.

The properties of the cross-junction are described with scattering matrix S_{ij}. Using clockwise numbering of ports we assume that input port is the port 1 then the port 4 is port for attachment of standards and loads under test. From symmetry of the device, the relation $S_{21} = S_{41} = S_{14} = S_{12} = S_{34} = S_{32}$ follows. Matched power meters are connected to the ports 1, 2, 3. Directional detectors are used as matched power meters, which measure voltages $U_{1,2,3}$ proportional to corresponding powers $P_{1,2,3}$ in the ports 1, 2, 3. Additionally directional detector for input power P_0 measurement is connected to the port 1. A device under test is attached to the port 4 since the connection to the port 3 results to symmetrical construction thus the identical signals are formed in the ports 2, 4. This fact involves ill-posed system of linear equation. Multiple reflections between device under test and transducer was taken into account by new variable A_4, which is bilinear transform of reflection coefficient Γ of the form $A_4 = \Gamma S_{41} / (1-\Gamma S_{44}) = A'_4+jA''_4$.

According to the technique [4] of processing in the holography with three reference signals the following system of two linear equations have to be solved

$$B\mathbf{a} = \mathbf{q}\,, \quad \mathbf{a} = \begin{bmatrix} A_4^{'} \\ A_4^{''} \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} q_{mn} \,/\, 2 \\ q_{kl} \,/\, 2 \end{bmatrix}, \tag{1}$$

where B is matrix with elements

$$B_{11} = \mathrm{Re}(\, S^*_{m1}S_{m4}\,) \mid S_{n4} \mid^2 - \mathrm{Re}(\, S^*_{n1}S_{n4}\,) \mid S_{m4} \mid^2,$$
$$B_{12} = \mathrm{Im}(\, S_{m1}S^*_{m4}\,) \mid S_{n4} \mid^2 - \mathrm{Im}(\, S_{n1}S^*_{n4}\,) \mid S_{m4} \mid^2,$$
$$B_{21} = \mathrm{Re}(\, S^*_{k1}S_{k4}\,) \mid S_{l4} \mid^2 - \mathrm{Re}(\, S^*_{l1}S_{l4}\,) \mid S_{k4} \mid^2,$$
$$B_{22} = \mathrm{Im}(\, S_{k1}S^*_{k4}\,) \mid S_{l4} \mid^2 - \mathrm{Im}(\, S_{l1}S^*_{l4}\,) \mid S_{k4} \mid^2.$$

Values qmn, which determine elements of the vector q, equals to

$$q_{mn} = (\, p_m - p_{0m}\,) \mid S_{n4} \mid^2 - (\, p_n - p_{0n}\,) \mid S_{m4} \mid^2, \tag{2}$$

with $p_{0i} = |S_{i1}|^2$ and $p_i = |S_{Li1}|^2\, U_{c0}U_i \,/\, (\, U_{ci}U_0\,)$, where S_{Li1} is equal to $S_{i1} + S_{i4}S_{11}\Gamma_c \,/\, (1-S_{11}\Gamma_c)$; Γ_c is a calibrating standard reflection coefficient, U_{c0} and U_0 are the voltages of input power under calibration and measurement correspondingly, U_{ci} and U_i are voltages being measured in cross-junction ports under calibration and measurement correspondingly.

According least square method system (1) transforms to

$$B^T B\, \mathbf{a} = B^T \mathbf{q}. \tag{3}$$

The condition number cond of matrix B can be calculated by standard way as the ratio of the maximum eigenvalue to minimum one. Thus, cond depends on frequency, it also depend on a combination of the detectors. Numerical experiment has shown that the solution of (3) coincides with the solution obtained according to the method of radical center with accurate to calculation error.

The advantage of the approach (3) with respect to the traditional method of the solution of the system of quadratic equations is the ability to use standard Tikhonov's regularization

$$(\, B^T B + \alpha I\,)\, \mathbf{a} = B^T \mathbf{q}. \tag{4}$$

Principle of generalized residual formalizes regularization parameter α searching as the root of equation

$$\| B\mathbf{a}^{\alpha} - \mathbf{q} \|^2 = (\, \delta + h\, \| \mathbf{a}^{\alpha} \|)^2 + \mu^2\,, \tag{5}$$

where δ is measure of error in vector $\mathbf{q}$, $h = \| B - B_r \|$ and B_{r} is rigorous operator, $\mu = \inf \| B\mathbf{a} - \mathbf{q} \|$ is measure of inconsistency. Values δ, h, μ can be determined by calibration procedure with three standard loads. It is very important that according to (5) α depends on value of A_4, thus α is function of load reflection coefficient. Principle of generalized residual demands to estimate norm of difference of rigorous operator and its approximation. That is rather difficult. The practical way to estimate α is to choose the best value of α for standard load with reflection coefficient that is approximately equal to reflection coefficient of load under test.

Based on the values of the scattering matrix of E-plane cross-junction of rectangular waveguides used in [5], we calculated the value of the condition number of the matrix B, some of which are given in the table 1. For angles of crossing waveguides less than 55 deg condition number increases dramatically, reaching at the angle of 15 degrees values of the tens of thousands, indicating the degeneracy of the matrix and, consequently, the significant coincidence of results of measurements in the measuring ports. Selected frequencies correspond to the beginning, middle and end of the operating range of the waveguide with section 7.2×3.4 mm, which allows to estimate the value throughout the frequency range. For intermediate frequency dependence of the condition number has smooth character without sharp deviation. The table 1 shows that the selection and combination of detectors in the formation of the matrix and right-hand vector of (1) are essential to ensure the stability of the solution, wherein the combination of $m = 1$; $n = 2$; $k = 2$; $l = 3$ provides significantly better results than the combination of $m = 1$; $n = 2$; $k = 1$; $l = 3$. An opportunity to use not the right angle, but the angle of 60 degrees allows you to change the layout scheme of the device without significant degradation of the accuracy of measurement. Further decreasing of the angle of waveguides connection involves unacceptable increase in complex reflection coefficient measuring error due to the rapid growth of the condition number of the matrix B.

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Table 1

The values of the condition number of the matrix B for some values of angles of crossing waveguides and frequencies

Frequency, GHz	Angle, deg						
	60	65	70	75	80	85	90
m = 1; n = 2; k = 1; l = 3							
26,0	26,62	16,24	11,01	8,01	6,12	4,86	3,98
32,0	19,20	9,35	5,98	4,34	3,42	2,84	2,45
37,5	68,23	176,21	5,71	9,48	91,91	10,72	5,01
m = 1; n = 2; k = 2; l = 3							
26,0	24,74	13,38	8,00	5,15	3,50	2,49	1,86
32,0	18,60	8,47	4,86	3,08	2,06	1,43	1,02
37,5	8,32	3,86	2,42	1,70	1,31	1,29	1,58

Conclusion. Changing the traditional method of processing measurement data for a 6-port meter on the basis of the E-plane cross-junction of waveguides by the method based on the principles of holographic recording with three reference signals, lets solve a system of linear algebraic equations. This fact opens up the possibility of a mathematical formulation of the problem, and, therefore, evaluation of construction using formal attributes such as the value of the condition number of the matrix. It is allowed to select the optimum combination of ports, which are connected with detectors. Starting from the condition number of allowable values, it was found that it is possible to change the waveguides cross-junction with right angles to a cross angle of 60 degrees, which allows to build a new compact construction of the meter.

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