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EVALUATION METHOD OF NONLINEAR DYNAMICAL PROCESSES CHARACTERISTICS AT TEMPORARY REALIZATIONS

Abstract. Evaluation methodology of nonlinear dynamical processes characteristics at temporary realizations that includes procedures of exact prediction estimation depth (correlation interval) and memory depth (attractor dimensions) for plotting mathematical model of large-sized crushing process is proposed.

Keywords: dynamical system, object of management, model, technological process, phase space

Introduction. The actual task is the identification and prediction of condition for management of hard management objects (MO) which are characterized by nonlinearity, instability and stochastic property. It allows to increase quality of management by means of increasing accuracy of estimation of their condition on the basis of mathematical model building for prediction of ore preparation processes (crushing and breakage). The main specialty of nonlinear crushing processes is presence of different operation modes that determines necessity of identification managed processes task solution in process of system functioning.

Research actuality. At present moment, determination methods of properties and characteristics of nonlinear dynamical systems that cause appropriate signals (temporary realizations) are proposed. At this device of nonlinear dynamic allows to estimate structural characteristics of dynamical model of nonlinear system [1].

Nonlinear process can be described with help of stream vector equation [2]:

$$\dot{x} = F(x, p), \tag{1}$$

or Poincare's discrete mapping:

$$x[k+1] = F\{x[k], p\}; x[k] = \{x_1[k], \ldots, x_{d-1}[k]\}, \tag{2}$$

where F – nonlinear dimension function d; x – coordinate vector; p – vectors of parameters of system order; k – unit of time $(t = k \cdot T)$; T – sampling period.

Dynamical systems (1) and (2) have four types of solution depending on values parameters of order p [1, 3]. System attractors as stable balance, limit cycle, quasiperiodic and chaotic are corresponded to these solutions.

Systems (1) and (2) lose condition steadiness (functioning mode) and transit into another condition during changing of parameter p. This transition is called as bifurcation [3].

For instance, such formal transition corresponds with results of theoretical and experimental research of ore large-sized crushing, as nonlinear dynamical MO with variable structure (dimension, dynamic mode) and parameters that depend upon ore properties, constructional and technological variables [4].

It is well known [1–3] that only with help of temporary realization we can determine entropy correlation, which characterized estimation of exact prediction of system condition, in what mode it situated, and correlation dimension of attractor (system order). Identification of such hard MO by traditional methods requires great expenses on experimental research. That is why it is effecttive to use methods of nonlinear dynamic.

Task assignment. The main purpose of the article is the development of characteristics estimation method of nonlinear dynamical process of ore preparation, also choice of sampling interval of generative process for mathematical model plotting.

Research results. Kolgomorov's entropy K is essential characteristic of motion in phase space of arbitrary dimension. It describes dynamical behavior on attractor and proportional to velocity of data losses about dynamical system condition in time. For regular motion K – entropy is equal to zero, for systems with determinate chaos – positive and constant. Entropy K is endless in case of system behavior as white noise, which means absence of process predictability.

Information value about system value is defined according to the formula

$$K_k = -\sum_{i_0 \text{K}\, i_k} P_{i_0 \text{K}\, i_k} \ln P_{i_0 \text{K}\, i_k}\ , \tag{3}$$

where $P_{i_0 \dots i_m}$ – combined probability that $x(t=0)$ is situated находится in spot i_0, $x(t=T)$ – в ячейкеin spot i_1,K , $x(t+kT)$ – in spot i_k. At this difference $K_{k+1} - K_k$ is additional data that is necessary for prediction in which spot i_{k+1} will be system if it earlier was situated in spots i_1K i_k.

Then K entropy is equal to:

$$K = -\lim_{T\to 0}\lim_{\varepsilon\to 0}\lim_{N\to\infty}\frac{1}{NT}\sum_{k=0}^{N-1}(K_{k+1}-K_k) = -\lim_{T\to 0}\lim_{\varepsilon\to 0}\lim_{N\to\infty}\frac{1}{NT}\sum_{i_0 \text{K}\, i_N} P_{i_0 \text{K}\, i_N} \ln P_{i_0 \text{K}\, i_N}\ , \tag{4}$$

where N – length of temporary realization; ε – spot size of phase space.

For estimation of K entropy by experimental data value of correlation entropy is used:

$$K_R = \lim_{\varepsilon \to 0} \lim_{k \to \infty} \ln \left[\frac{R_k(\varepsilon)}{R_{k+1}(\varepsilon)} \right] \le K , \tag{5}$$

where $R_k(\varepsilon) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j} \upsilon \left[\varepsilon - \left\| x_i - x_j \right\|_k \right] = \sum_{i_1 \mathrm{K}\, i_k} P^2_{i_1 \Lambda\, i_k}$ – generalized correlation integral; $\sum_{i,j} \upsilon \left[\varepsilon - \left\| x_i - x_j \right\|_k \right]$ – number of pairs i and j, for which distance $\left\| x_i - x_j \right\|_k \le \varepsilon$; $\left\| x_i - x_j \right\|_k = \sqrt{\sum_{n=0}^{k-1} \left(x_{i+n} - x_{j+n} \right)^2}$; υ – Heaviside step function, $x_i = x[iT]$.

Additionally K entropy allows to determine average time for which can predict system condition. At this, exist prediction is possible only on time interval T_{pr}, $\varepsilon \cdot e^{KT_{np}} = 1$ [2], then

$$T_{np} = \frac{1}{K} \ln \left(\frac{1}{\varepsilon} \right). \tag{6}$$

Estimation of predictability interval on research data is executed similarly to expression (6):

$$T_{Rnn} \frac{1}{K_R} \ln \left(\frac{1}{\varepsilon} \right) \ge T_{np} . \tag{7}$$

Therefore, T_{Rnn} is determined depth of exist prediction of MO condition on value of correlation interval predictability.

At this required deflection interval (prediction depth) is determined by value of discretization period T and equivalent delay time τ of system.

Then with taking into account (7) must be execute condition

$$T + \tau \le T_{Rnn} , \tag{8}$$

from which corresponding value of discretization process is chosen. Here equivalent delay time τ can be determined on maximums of intercorelation functions of entrance and output system coordinates.

The other way of period T choice is statistical approach [5], according to which with favorable for practice accuracy has to be performed following condition

$$T + \tau \le 0{,}2 \cdot \tau_{кор} , \tag{9}$$

where $\tau_{кор}$ – correlation interval of outlet system coordinate.

To reach the fulfillment of condition (9) is possible by means of sequential increasing of period T that leads to tapering of average process spectrum towards initial and, consequently, to widening of its autocorrelation function (i.e. increasing of average process of correlation interval) [5].

Correlation dimension characterizes hardness of dynamical system attractor, i.e. characterizes minimal number of counting (memory depth) of varieties that entered in the mathematical model of system.

Distance between near points of attractors before and after bifurcations describes Hausdorf's fractional dimension D which is number that characterized velocity of spots of cover growing during decreasing of spots size:

$$D = -\lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(\varepsilon)}, \tag{10}$$

where $N(\varepsilon)$ – number of spots.

Hausdorf's D dimension estimation, both as entropy K estimation can be made on experimental research [2].

Let the dynamical system path be described as $x(t) = [x_1(t), \ldots, x_d(t)]$ and d-phase space is divided on spots with size ε^d. Thus, entering probability of point that belong to attractor in i-spot $(i = 1,2,\ldots N(e))$ is calculated as:

$$p_i = \lim_{N \to \infty} \frac{N_i}{N}, \tag{11}$$

where N_i – number of points in this spot.

Numerical dimension D estimation executes according to Grassberg-Procaccia algorithm of correlation dimension calculation:

$$D_R = \lim_{\varepsilon \to 0} \frac{\log\left(\sum_{i=0}^{N(\varepsilon)} p_i^2\right)}{\log \varepsilon} = \lim_{\varepsilon \to 0} \frac{\log R(\varepsilon)}{\log(\varepsilon)} \le D, \tag{12}$$

where $R(\varepsilon) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j} \upsilon\left[\varepsilon - \left\| x_i - x_j \right\|\right]$ – correlation integral; $\sum_{i=0}^{N(\varepsilon)} p_i^2$ – probability that two points of attractor lay inside the spot ε^d (possibility that two points of attractor separate of less ε).

Dependence $\log R(\varepsilon)$ from $\log \varepsilon$ is constructing and realizing linear unit search for determining D_R, incline of which determines target value of correlation dimension D_R for further determining of attractor's dimension.

Dimension d in phase space and correlation dimension D_R do not change and is minimal dimension of attractor's outlay, i.e. the smallest full dimension of phase space that contains full attractor. Therefore, dimension of phase space d is determines as $D_R(d)$.

However, as a consequence of theorem about enclosure it is known that dimension estimation of phase space d is determined through the attractor's dimension D_R as:

$$d \geq 2D_R + 1 . \tag{13}$$

During calculation of correlation dimension D_R one of the problem was choice of experimental data N values volume and discretization period T [3]. At this, it is effectually to take into account size of temporary window $T_H = NT$ and take into consideration presence of limit on determining of correlation dimension D_R maximal value,

$$D_{R\max} = \frac{2\lg N}{\lg(\varepsilon_{\max}/\varepsilon)}, \tag{14}$$

which means that calculation algorithm of system dimension can't give much value than $D_{R\max}$ at given number of points N, where $\varepsilon_{\max}$ – attractor size.

Therefore, calculation of D_R dimension allows to reconstruct attractor and to determine dimension d of mathematical model MO variables (memory depth of source and target variables of model).

Experimental results. Characteristics estimation was executed by means of modeling process of large-sized breakage (LSC) of ores [6], whereas output coordinate of LSC process of ores was considered temporary realization of class content + 100 mm in broken ore G_{+100}. (Figure 1).

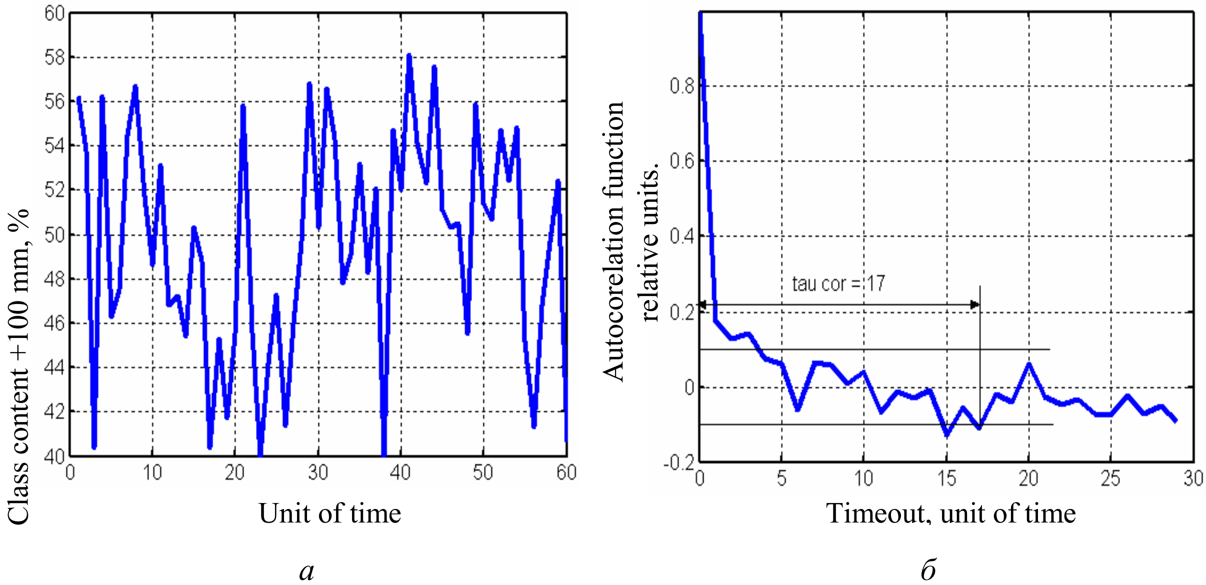
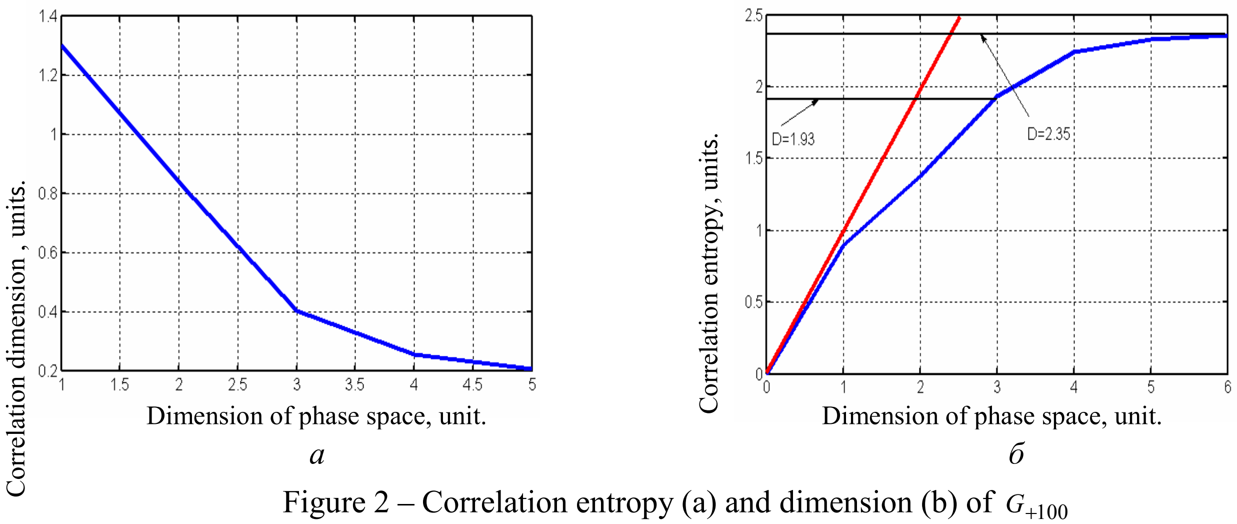


Figure 1 – Temporary realization of signal G_{+100} (a) and its

correlation function (b)

As a result for signal G_{+100} were determined correlation entropy (Figure 2, a) and dimension (Figure 2, b), its value accordingly $K_R = 0.405$ and $D_R = 1.931$.



а *б*

Figure 2 – Correlation entropy (a) and dimension (b) of G_{+100}

At this correlation interval of process predictability (depth of exist prediction) that causes signal G_{+100} according to expression (7) and is equal to $T_{Rnn} = 3.97$ clock time.

The main specialty of LSC is that ore on breakage is delivered in casual moments of time that allow to use sequence numbers of portions as measure. As required, transition to real time can be execute through the average time interval of portions of entries.

For delay time process τ estimation were calculated intercorrelation functions of process LSC output on channels of average coarseness G_{en} and hardness K_p of entered ore and also on management channel and relieve slot of crushing C (Figure 3).

From Figure 3 it is following that according to maximums of intercorrelation functions, delay is equal to $\tau = 2$ clock time. Thus, value of discretization process $T = 1$ (unit of time is one portion of ore with average time on entry for crushing), because condition is executed on expression (8) $1 + 2 \leq 3.97$ units of time and executing condition on expression (9): units of time (here value of correlation interval $\tau_{cor} = 17$ units of time at significant value of correlation connection 0.1).

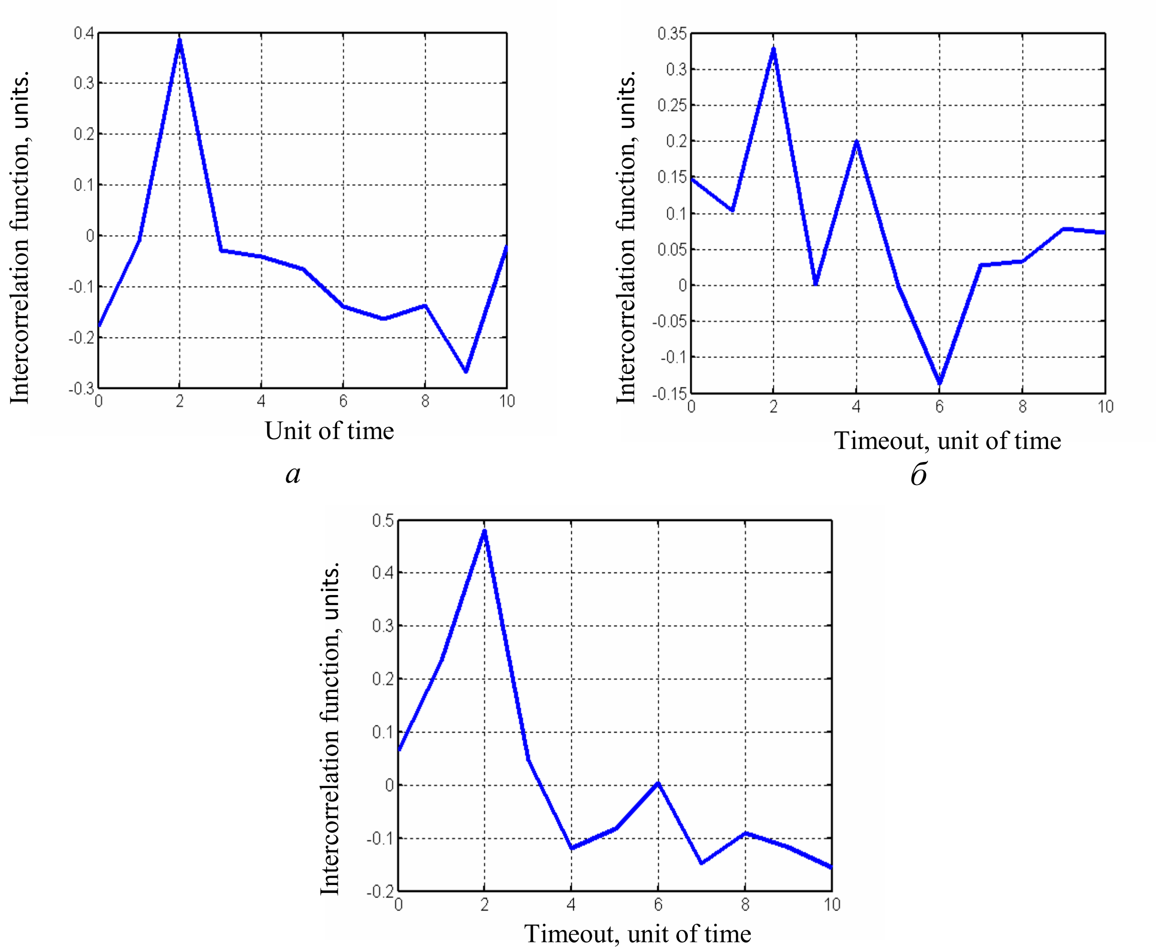


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Figure 3 – Intercorrelation functions on channels: $G_{+100} - G_{en}$ (a), $\Gamma_{+100} - Kp$ (b) and $G_{+100} - C$ (c)

For determining dimension of phase space attractor (memory process depth) inputs was calculated its estimation above the expression (13), but estimation of value d below was determined with help of Figure 2, b. From Figure 2, b it can be seen that dimension D_R practically stops to increase during dimension of phase space $d \geq 3$. In consideration of the foregoing, we get $3 \leq d \leq 5$.

Therefore, for intensification LSC process solution that causes signal G_{+100}, depth of exist prediction is equal to 4 units of time, but memory depth is equal from 3 to 5 units of time.

Conclusions. Estimation method of nonlinear dynamical processes characteristics on temporary realizations that include estimation procedures of entropy and dimension and choice of interval of discretization process. It allows to estimate prediction interval (depth of exist predict) and dimension (memory depth) of mathematical model of large-sized crushing process.

Further research has to be directed on development of data technology of mathematical models of identification of nonlinear dynamical processes with usage of proposed estimation method of its characteristics.

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