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NONLINEAR RECURRENCE ANALYSIS IN TREATMENT OF TIME SERIES

Abstract. In the article basis foundation and possibilities of treatment of time information by the help of recurrence analysis are considered. Directions of a development of the recurrence analysis and it use for the real processes are specified.

Key words: recurrence analysis, simulation, time series.

Introduction

A simulation on the time series got in the process of experiment (on the discrete sequences of information) is one of actively developing modern directions of a mathematical investigation. At the study of the difficult natural phenomena, a time series often is a unique information present at a researcher. Therefore the most complete opening of the properties of the explored process contained in observed data, is one of conditions of successful construction of a model. From other side, the research of time series acts important rule in work with the already built mathematical models. As a rule it is tasks related to the selection of coefficients of equations, a verification of equations, a study of properties of model. It is especially actual for the design of the dynamic systems with the chaotic behavior and more precisely nonlinear dynamic systems.

A correct construction of mathematical model depends on that as far as complete the basic data analysis was executed, the properties of the studied object (process) are exposed and estimated. The correctness of the built model depends on that as far as complete a model was analyzed. In the case of exposure of insufficient adequacy of model a process repeats oneself a new from one of stages, that requires additional temporal, natural and other expenses. Conventionally that there is adequacy of prognosis of behavior of the initial real system at the dynamic design by the basic criterion of estimation of rightness of a mathematical model. Presently facilities of analysis of time series are one of major instruments of researcher at the simulation.

Since 1981 year the set of traditional (linear) methods of research of time series was substantially extended by the nonlinear methods derived from the theory of nonlinear dynamics and chaos; many researches were devoted to estimation of nonlinear descriptions and properties of the natural and artificial systems. Most methods of nonlinear analysis require either long enough or stationary series of information. However it is far not always can be obtained at practical research of the real systems. Moreover, as it were showed by Manuca [1] and Savit [2], that these methods gave satisfactory results even for the idealized models of the real systems.

The following stages of mathematical simulation, which on the whole are taken to such execution sequence, are known: 1. The obtaining and analysis of the observed time series (basic data) after for raising of a problem; 2. The description of analytical model; 3. The choice of structure of the model: the type of equations, the type of functions, the establishment of communication of variables with the observed magnitudes; 4. The tuning of model: calculation (selection) of parameters; 5. The verification of model (a check of its adequacy); 6. The application for the analysis of the concrete real process.

Simulation on the basis of time series

Lets the results of measuring of the real process are represented by the time series $\{\vec{u_i}\}_1^n \equiv \{\vec{u_1}, \vec{u_2}, \dots, \vec{u_n}\}\{\vec{u_i}\}_1^n \equiv \{\vec{u_1}, \vec{u_2}, \dots, \vec{u_n}\}$, where $\vec{u_i} = \vec{u}(t_i), t_i = i\Delta t,$ $\vec{u_i} = \vec{u}(t_i), t_i = i\Delta t,$ n is a number of observed data; Δt Δt is an interval of time between measurements. More often we have 1-D series because as rule there is no information about all variables or not possibilities of them to measure. Further, the analysis is executed and verbal description of model taking into account obtained information from a time series and also other a priori information known to us before is carried out. Assume that the mathematical model is created; it can be the a finite determined model of difference equations of kind $\vec{x}_{n+1} = \vec{F}(\vec{x}_n, \vec{c})\vec{x}_{n+1} = \vec{F}(\vec{x}_n, \vec{c})$, the ordinary differential equations of kind $\dot{\vec{x}} = \vec{F}(\vec{x}, \vec{c})\dot{\vec{x}} = \vec{F}(\vec{x}, \vec{c})$, here $\vec{x}$ $\vec{x}$ is a measured vector of the state, F F is the vector-function, $\vec{c}\vec{c}$ is a measured vector of parameters, n n is a discrete time for the first model, and in the second case it is a continuous time. On this stage the choice of type and number of equations is carried out, the type of functions F F incoming in equations and number of variables is determined (components of vector x x). We will mark that variables the observed magnitudes can be chosen $\vec{x} \equiv \vec{u}\vec{x} \equiv \vec{u}$, but connection between them can take place in general case $\vec{u} = \boldsymbol{h}(\vec{x})\vec{u} = \boldsymbol{h}(\vec{x})$, where $\boldsymbol{h}(x)\boldsymbol{h}(x)$ is a measuring function. Further, it is required to carry out the choice of values of parameter for tuning of model. Finally, to carry out verification of the built model that verification of its adequacy to the real process. If this model dissatisfies to this requirement than it is finished off taking into account the got results.

Thus, it is visible that facilities of analysis of time series take important place on the stages of empiric construction of mathematical model and act large part in the receipt of high-quality results. The development of nonlinear dynamics theory and chaos brought in understanding of advantage of nonlinear essence of the natural phenomena and the design the last years is carried out mainly with the use of nonlinear difference and differential equations of a different dimension. The applied methods of nonlinear analysis appeared of the little uses for researches of initial time series. On the derived time series $\{\vec{u_i}\}_1^n$ $\{\vec{u_i}\}_1^n$ as a result of observations it is possible to judge about properties and behavior of the system already simply building the graphic image of trajectory in corresponding state space (the periodic or chaotic systems have the portraits of a particular kind). However, if dimension is more than 3 such analysis is very difficult, because it becomes a necessity to do projections in two and three-dimensional subspaces. The connection of results of observations with state variables and model equations in the general case complicated yet and by noises. For example, fort 1-D map $x_{n+1} = F(x_n + \xi_n, c)$ $x_{n+1} = F(x_n + \xi_n, c)$ and $u_n = x_n + \zeta_n$ $u_n = x_n + \zeta_n$, where $\xi_n \xi_n$ is a dynamics noise (it is a noise influencing on the dynamics of the system), and $\zeta_n \zeta_n$ is a measuring noise (it the noise influencing on the results of measurements); the exact decision of the problem about finding of vector of parameters is possible only in ideal case $\xi_n = \zeta_n = 0$ $\xi_n = \zeta_n = 0$ that in the real researches is the practically unrealized scenario.

Deprived of the indicated defects and one of the most interesting modern methods is a recurrence diagrams methods getting in the last decade wide theoretical development and practical confession. A method is based on fundamental property of the dynamic systems, marked yet at the end of 19th age by the prominent French mathematician Henry Poincare and formulated as the recursiveness theorem: "If the system takes the dynamics which happen in a bounded subset of the state space, then this system almost for certain (with probability practically to equal unit) as much as desired nearly goes back to some initially behavior".

The recurrence behavior is periodicity or irregular cyclicity, it is not only the natural systems, but also complete systems created by a man. A recursiveness (repetition) states in sense of passing of the last section of a trajectory in the state space near enough to the previous section is fundamental property of the dissipative dynamic systems.

The idea of reconstruction of attractor arises up to the Takens's theorem [3] with the help of which it is possible to recover the state space of the attractor of system and to make the present of dynamics of all system on the change of one variable.

Consider the dynamic system (DS) $\varphi^t(x) \varphi^t(x)$ with the state space M, where M is the compact d-measured real variety (dim M = d). Assume that numbers formative a time series are the values of some observed scalar function of the state of DS $x(t)$: $x_i = h(x(t_i))$ and these numbers are the sequence of the measured instantaneous values of variable $x(t)$. Then it is possible to represent this sequence in the m-measured space so that every value of this time series of x(ti) was represented by a point of this space with coordinates $\{x(t_i), x(t_i + ^{\tau\ \tau}), \ldots, x(t_i + (m-1)^{\tau\ \tau})\}$. We will name this m-measured space by a space of embedding and the set of points designing initial attractor we also will name by a reconstructed attractor. By $\tau\ \tau$ denote a temporal step between of elements of the time series and the vector $[x(t]_i) [x(t]_i)$ we will designate by x_i . Then

$$x_{i+1} x_{i+1} = \varphi^{\tau}(x_i), x_{i+2} \varphi^{\tau}(x_i), x_{i+2} = \varphi^{2\tau}(x_i) \varphi^{2\tau}(x_i), \ldots, x_{i+m-1} x_{i+m-1} = \varphi^{(m-1)\tau}(x_i)$$
$$\varphi^{(m-1)\tau}(x_i).$$

Construct the vectors $z_i z_i = \{x_i, x_{i+1}, \ldots, x_{i+m-1} x_i, x_{i+1}, \ldots, x_{i+m-1}\}$, where

$$x_i = h(x_i),$$
$$x_{i+1} = h(x_{i+1}) = h(\varphi^{\tau}(x_i)),$$
$$x_{i+2} = h(x_{i+2}) = h(\varphi^{2\tau}(x_i)),$$
$$\dots$$
$$x_{i+m-1} = h(x_{i+m-1}) = h(\varphi^{(m-1)\tau}(x_i)).$$

All components of vector $z_i z_i$ are connected with the same state of DS $x_i x_i$. Then there exists a vector-function V such that this function maps vectors $x_i x_i$ into points of the the m-measured space $R^m R^m$, $z_i = V(x_i)$, $x_i \in M$, $z_i \in R^m$. $z_i = V(x_i)$, $x_i \in M$, $z_i \in R^m$.

The Takens's theorem asserts that a typical property of the map V is that at $m \geq 2d+1$ $m \geq 2d+1$ this map is embedding M into $R^m R^m$.

By $S = V(M) S = V(M)$ denote an image M into $R^m . R^m$.

According to the Takens's theorem in typical case this image must not have self-intersections. The function V has the inverse function $V^{-1} V^{-1}$ defined on S. Its image in the z-space corresponds to every trajectory DS. Thus these images have those properties that initial trajectories. On S it is possible to define the dynamic system

$$x_i = V^{-1}(z_i), x_{i+1} = \varphi^{\tau}(x_i),$$
$$z_{i+1} = V(x_{i+1}) = V(\varphi^{\tau}(x_i)) = V(\varphi^{\tau}(V^{-1}(z_i))) \equiv P(z_i).$$

where P acts only on S.

Thus, we have two maps:

$x_{i+1} x_{i+1} = \varphi^{\tau}(x_i) \equiv \Phi(x_i), \Phi: M \to M$ $\varphi^{\tau}(x_i) \equiv \Phi(x_i), \Phi: M \to M$,

$z_{i+1} = P(z_i)$, $P: S \to S$. $z_{i+1} = P(z_i)$, $P: S \to S$.

These maps can be considered as the mappings connected by the convertible replacement of variables $z = V(x). z = V(x)$. The properties which are invariant with respect to such replacement must coincide for the systems. To these properties, in particular, belongs a correlation dimension which it is possible to define from experimental data not knowing all variables of the dynamic system. The S and P properties depend on the dynamic system φ φ, the observed function h, the delay τ τ and the dimension of embedding m.

Grassberger and Procaccia method [4] consists in reconstruction of attractor which like initial, by a successive shift on the magnitude τ τ. For estimation of dimension of embedding consistently get new dimensions and measure some characteristic of turning multidimensional series. After thus some value this magnitude stops to be increased that speaks about achievement of dimension of embedding. For check such achievement of dimension a correlation integral $C(\varepsilon) C(\varepsilon)$ is used in our paper.

A correlation integral (correlation index) is probability of that a time series contains the pair of points such that the distance between these points does not exceed ε ε . The calculation of correlation integral is produced under the formula:

$$C(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1, i \neq j} \theta(\varepsilon - |x_i - x_j|)$$

where $\theta(x) \theta(x)$ is Heaviside function: $\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$, N $\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$, N is a number of observations, ε ε is distance, x_i, x_j x_i, x_j are elements of selection.

The magnitude

$$D_c = \lim_{\varepsilon \to 0} \frac{\log C(\varepsilon)}{\varepsilon}$$

is called the correlated dimension.

In order that to estimate $D_c D_c$ it is possible by linear approximation. At enough small ε ε we have.

$\frac{\lim_{\varepsilon \to 0} \left(C(\varepsilon) \right)}{\varepsilon} \approx \varepsilon^{D_c}$ **From here it follows that** $\log C(\varepsilon) = D_c \log \varepsilon + const,$ $D_c = \frac{\log C(\varepsilon) - const}{\log \varepsilon}$.

A correlation integral can be computed for points in initial state space and for the reconstructed vectors. In the second case a correlation dimension becomes a function not only from ε ε but also from the parameters of reconstruction of m and τ τ .

The dependence on two last parameters allows to diagnose chaotic, a level of noise, a time of predictable.

As it was already marked the practical realization of ideas of reconstruction often collide, from one side, with the problem of boundedness of time series, from other side, with the problem of stationary of the explored object.

For example, cardiogram which take off long time, is not a stationary process. The requirement of stationary of process it is possible to consider practically observed within the limits of section cardiogram by duration of to 1 sec.

In further researches offered in 1987th by Eckmann, Kamphorst and Ruelle recurrence diagrams, based on this fundamental property, and is allowed to represent the phase trajectory of any dimension on a 2-D binary square matrix, size of which is determined by length the time series through the property of recurrentness. Besides, visual possibilities, there is the method of quantitative analysis of the structures formed on the image of recurrence diagram. The modern researches were shown that a recurrence diagram contains all necessary information about the dynamics of the system. Due to works of Joe Zbilut, Norbert Marvan, Marco Thiel, Carmen Romano and others the possibilities of this method were substantially enriched in the last decade. In spite of high enough interest to this method from the side of foreign scientists (the amount of publications on his application in scientific activity makes hundred works), it application rarely enough meets in domestic scientific and technical practice. A tool which would unite in itself all last achievements in area of recurrence analysis here would be oriented to creation on his basis of complexes of the programs is absent.

Often a researcher is forced to do the arbitrary choice of managing parameters of quantitative analysis because there does't exist of criterions for their choice. Sometimes researchers assume bringing of strange, not based on the recurrence theory, methods with the purpose of attempts of «reconstruction» of parts of diagrams that brings in the analysis the information not got directly from the explored information. Also it should be noted that the on itself recurrence analysis is the rich field for researches, both method and aspects of his application. The method of recurrence diagrams is complex method of analysis of time series not demanding to quality of input information, combining in itself visual possibilities (diagrams) and powerful numeral tools (measures). Nevertheless, a method on itself represents the field for researches, including study of possibilities and features of application in practice of construction of mathematical models. A flexible enough tool for the use at construction of the problem-oriented programs is absent (in particular, oriented to the regular applied use). Consequently, development and research of methods and algorithms of application of recurrence diagrams on the stages of mathematical design, and also development of flexible tool realizing possibilities of the method, are the actual problem of the day.

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