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СРАВНИТЕЛЬНЫЙ АНАЛИЗ ИНФОРМАЦИОННОЙ СЛОЖНОСТИ ХАОТИЧЕСКИХ И СТОХАСТИЧЕСКИХ ВРЕМЕННЫХ РЯДОВ

Abstract. The comparative analysis of the statistical properties of realizations of chaotic and stochastic processes having different correlation structure: uncorrelated noise, autoregressive processes with short-term dependence and fractal processes with long-term memory. Depending on complexity measures of time series of process parameters were obtained. The time series corresponding to a variety of complex dynamical systems were investigated.

Keywords: time series, measures of complexity, approximate entropy, recurrence plot, pseudo-phase space, embedding dimension.

INTRODUCTION AND OBJECTIVE

Mathematical models of complex systems exhibiting irregular dynamics are both random and deterministic chaotic processes. One of the objectives of time series analysis is to extract information from the series and infer the properties and mechanism of the process that generates the series.

There are many approaches to the study of time series based on traditional statistical analysis, and the methods of nonlinear chaotic dynamics. Most methods of chaotic dynamics used for time series analysis, based on the reconstruction space of single realization using the procedure Packard-Takens [1,2]. The reconstruction of the pseudo-phase space allows us to compute the embedding dimension, which is the main means of distinguishing chaotic and random processes [1-3]. This approach allows us to well distinguish between chaotic dynamics and uncorrelated random noise, however, it has no effect for the fractal random processes having long dependence. [4-6].

The characteristics of the complexity of the system behavior are entropy and recurrence measures [7]. The method of recurrence plots is based on the fundamental property of dissipative dynamical systems - recurrence states. This method of analysis, based on the representation of process properties in the form of geometric structures, is a means for detection the hidden dependencies in the observed processes [7-9]. Numerical analysis of recurrence plots allows us to calculate the measure of complexity structures of recurrence plots, such as a measure of recurrence and determinism etc [9,10]. The characteristic of the complexity of the system behavior is entropy. Entropy and recurrence methods of time series analysis are based on the reconstruction space of single realization using the procedure Packard-Takens [4,7-9].

The purpose of this work is to conduct a comparative recurrence and entropy analysis of deterministic chaotic and random self-similar realizations to identify the mechanism, which generates researched series.

RESEARCH METHODS

Construction of pseudo-phase space [1,2]. The main idea of the application of nonlinear dynamics methods to the analysis of the realizations of a dynamical system is that the basic structure, which contains all the information about the system, namely, an attractor of a system, can be reconstructed by measuring only single component of this system Widely used procedure Packard-Takens allows to restore the phase trajectory of a dynamical system from single realization:

$$F(t) = [x(t), x(t+\tau), ..., x(t+m\tau)], \tag{1}$$

where $F(t)$ – m - dimensional pseudo-phase space, $x(t)$ – time realization, τ – delay period.

The construction of recurrence plot [7-10]. Recurrence plot is a projection of the m-dimensional pseudo-phase space onto the plane. Let point x_i corresponds to the phase trajectory $x(t)$ describing the dynamic system in the m-dimensional space at a time $t = i$, for i = 1, ..., N, then the recurrence plot RP is array of pixels, where a nonzero element of the coordinates (i, j) corresponding to the case where the distance between x_j and x_i is smaller ε:

$$RP_{i,j} = \Theta(\varepsilon - \| x_i - x_j \|), \;\; x_i, x_J \in R^m, \, i, j = 1,...N, \tag{2}$$

where ε – size neighborhood of the point x_i; $\left\| x_i - x_j \right\|$ – distance between points; $\Theta(\cdot)$ – Heaviside function.

Analysis of the plot topology allows us to classify the observed processes: homogeneous processes with independent random values, processes with slowly varying parameters, periodic or oscillating processes corresponding to nonlinear systems, etc. Numerical analysis of recurrence plots allows us to calculate the measure of complexity structures of recurrence plots, such as a measure of recurrence and

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determinism etc. The measure of recurrence RR shows the density of recurrence points: $RR = \frac{1}{N^2}\sum_{i,j}^{N} RP_{i,j}$, where N – total number of points.

Measure of determinism Det is a characteristic of predictability process and equal to the ratio of the number of points in diagonal lines to the total number of recurrence points: $Det = \sum_{l=l_{\min}}^{N} P\ (l) / \sum_{i,j}^{N} RP_{i,j}$, where l_i - length of the i -th diagonal line; $P\ (l) = \{l_i, i = 1,..., N_l\}$ - frequency distribution of the diagonal lines lengths; N_l - number of diagonal lines.

The calculation of approximate entropy [4,7,11]. Aproximate entropy *ApEn* is the statistics of time series regularity that defines the possibility of its forecasting Consider a time series $\{x_i\}, i = 1,..., N$. Let the vector $P_m(i)$ is subsequence values $\{x_i, x_{i+1},..., x_{i+m}\}$ length of *m*. Two vectors $P_m(i)$ и $P_m(j)$ will be similar, if the following condition $\left|x_{i+k} - x_{j+k}\right| < \varepsilon,\ 0 \le k < m$. For each $i = 1,..., N - m + 1$ value $C_{im}(\varepsilon)$ is calculated $C_{im}(\varepsilon) = \frac{n_{i\,m}(\varepsilon)}{N - m + 1}$, where $n_{i\,m}(\varepsilon)$ is number of vectors, that similar vector $P_m(i)$.

Approximate entropy *ApEn* determined by the formula

$$ApEn(m, \varepsilon) = \ln \frac{C_m(\varepsilon)}{C_{m+1}(\varepsilon)}, \tag{3}$$

where $C_m(\varepsilon) = \frac{1}{N - m + 1} \sum_{i=1}^{N-m+1} C_{i\,m}(\varepsilon)$.

MODEL DATA

Chaotic realizations [1,2]. Chaos is a complex dynamics of a deterministic systems in steady state. The main feature of such systems is sensitive dependence to arbitrarily small changes in initial conditions. If d_0 is the initial distance between two points, then short time t later the distance between the trajectories, which start from these points, becomes $d(t) = d_0 e^{\lambda t}$, where the value of λ is the Lyapunov exponent. This leads to the loss of deterministic predictability and the need to introduce probabilistic characteristics to describe the dynamics of chaotic systems. Iterated maps $x_{n+1} = f(C, x_n)$, where C is control parameter, are the most simple and intuitive mathematical chaotic models.

For a wide class of nonlinear functions f the sequence $\{x_n\}_{n=0}^{\infty}$ is chaotic Logistic map is the most famous example of chaotic maps:

$$x_{n+1} = Ax_n(1 - x_n), \tag{4}$$

where A – control parameter, $A \in (0,4]$ and $x_n \in [0,1]$.

Realizations of an autoregressive process [3]**.** As processes with short-term dependence chosen autoregressive process of order 1:

$$X(t) = \phi X(t-1) + \varepsilon(t), \tag{5}$$

where $\varepsilon(t)$- uncorrelated white noise; ϕ– autoregressive coefficient, $|\phi| < 1$. Autoregressive coefficient value ϕ characterizes the degree of the autocorrelation process.

Stochastic self-similar realizations [3,12]**.** Stochastic process $X(t)$ is self-similar with self-similarity parameter H, if the process $a^{-H}X(at)$ is described by the same finite-dimensional distributions that $X(t)$. One of the most famous and simple models of stochastic dynamics that have fractal properties, is the fractional Brownian motion (FBM).

FBM with the parameter $H = 0,5$ coincides with the classical Brownian motion. Parameter H called the Hurst exponent, is the degree of self-similarity. Along with this property, the index H characterizes the measure of long-term dependence of a stochastic process, i.e. that autocorrelation function $r(k)$ decreases as a power law.

RESULTS OF RESEARCH

Carried out recurrent analysis detected strong differences in visual topology and the numerical characteristics of realizations of the above processes. We first consider the example of a completely different process on complexity: a periodic motion and uncorrelated white noise. It is obvious that the characteristics of chaotic and random processes must be located within the range of characteristic values calculated for the periodic and completely random trajectories. Table 1 shows the corresponding values of the measures of recurrence *RR*, determinism *Det* and approximate entropy *ApEn*. To construct the plot and calculate the characteristics in this case the length of realizations was chosen $N = 1000$.

Table 1. Quantitative characteristics of complexity of sinusoid and uncorrelated noise

	RR	*Det*	*ApEn*
Sinusoid	0.18	0.998	0.03
Uncorrelated noise	0.0003	0.025	1.7

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Table 2 shows the mean of recurrence *RR*, determinism *Det* and approximate entropy *ApEn* corresponding to the realizations of map (4) for the control parameter A=3.7, 3.9, 4 (Lyapunov exponent is equal to λ=0.37, 0.5, 0.69); autoregression realizations (5) when the values ϕ=0.3, 0.6, 0.9; FBM realizations when the Hurst exponent H=0.3, 0.6, 0.9. Greater value of Lyapunov exponent corresponds to a greater randomness of the system In each case, the values *RR* and *Det*, as a measure of regularity, decreases, and the entropy *ApEn* increases with randomness or uncorrelation.

Table 2. Quantitative characteristics of complexity of realizations

Logistic map				Autoregression				FBM			
A	*RR*	*Det*	*ApEn*	φ	*RR*	*Det*	*ApEn*	*H*	*RR*	*Det*	*ApEn*
3.7	0.008	0.1	0.93	0.3	0.0003	0.03	1.72	0.3	0.02	0.55	0.47
3.9	0.004	0.07	1.2	0.6	0.0005	0.05	1.65	0.6	0.02	0.87	0.21
4	0.002	0.05	0.86	0.9	0.002	0.13	1.25	0.9	0.01	0.95	0.12

In this work the time series corresponding to a variety of complex dynamical systems. In particular, the RR-intervals series were investigated. RR-interval is the time interval between adjacent teeth of electrocardiogram and it equals to the duration of the cardiac cycle. As an example of financial series, the dynamics of change in the index S&P500 for 2004-2008 was examined. Quantitative recurrence and entropy characteristics obtained from the time series are presented in Table 3.

Table 3. Quantitative characteristics of complexity of time series

	RR	*Det*	*ApEn*
RR-intervals	0.18	0. 84	1,87
S&P 500	0.06	0.91	2.1

Based on the results of qualitative and quantitative analysis can be propose for modeling realizations RR-intervals to use deterministic chaotic systems, while the mathematical modeling of S&P500 series should be based on self-similar stochastic processes. For an correct choice of the model in the first case the estimation of such characteristics as the Lyapunov exponent, invariant measure distribution, etc. is necessary, and in the second case – the estimation of fractal characteristics.

CONCLUSION

The comparative analysis of the statistical properties of realizations of chaotic and stochastic processes having different correlation structure: uncorrelated noise, autoregressive processes with short-term dependence and fractal processes with long-term memory. The dependences of information complexity measures of time series, such as a measure of recurrence, a measure of determinism, entropy, scaling, etc., from the parameters of the processes: the bifurcation parameter, the Hurst exponent, autoregression coefficient. In this work the time series corresponding to complex dynamical systems: electrical biosignals and financial structures. Based on the results of the analysis mathematical model, taking into account the correlation and recursive structure of the time series were proposed.

REFERENCES

1. Мун Ф. Хаотические колебания / Ф. Мун. – М.: Мир, 1990. –304 с..
2. Шустер Г. Детерминированный хаос: Введение / Г. Шустер. – М.: Мир, 1988. –240 с
3. Ширяев А. Н. Основы стохастической финансовой математики / А. Н. Ширяев. – М.: Фазис, 1998. – Т. 1: Факты. Модели. – 512 с.
4. Pincus S.M. „Approximate entropy as a measure of system complexity” / S.M. Pincus. Proc. // Natl. Acad. Sci. Vol.88, pp. 2297-2301
5. Dabi-Prashad O. Investigation of Time Series of Original Values of Currency Rates Measured on Small Time Frames on FOREX Using Methods of Chaos Theory / O. Dabi-Prashad, L. Kirichenko // Radioelectronics & Informatics. – 2009. – №4. – P. 18–24.
6. Кириченко Л.О., Кобицкая Ю.А., Хабачева А.Ю. Сравнительный рекуррентный анализ хаотических и случайных процессов / Л.О. Кириченко, Ю.А. Кобицкая, А.Ю. Хабачева //Physical and technological problems of radio engineering devices, telecommunication, nano- and microelectronics Proceeding of the III International Scientific-Practical Conference October 24–26, 2013 Chernivtsi, Ukraine.
7. Дербенцев В.Д., Сердюк О.А., Соловйов В.М., Шарапов О.Д. Синергетичні та еконофізичні методи дослідження динамічних та структурних характеристик економічних систем / В.Д. Дербенцев, О.А. Сердюк, В.М. Соловйов, О.Д. Шарапов // Монографія. – Черкаси: Брама-Україна, 2010. – 287 с.
8. Eckmann J.P. Recurrence Plots of Dynamical Systems / J.P. Eckmann, S.O. Kamphorst, D. Ruelle // Europhysics Letters 5. – 1987. – P. 973-977.

 ISSN 1562-9945

9. Marwan N. Recurrence-plots-based measures of complexity and application to heart-rate-variability data / N. Marwan, N. Wessel, U. Meyerfeldt, A. Schirdewan, J.Kurths // Physical Review, E66,– 2002.
10. Zbilut J.P., Zaldivar-Comenges J.-M., Stozzi F. Recurrence quantification based Liapunov exponent for monitoring divergence in experimental data // Phys. Lett. A, V. 297, 2002. – pp. 173-181.
11. Joshua S., Richman J., Moorman R. Physiological time–series analysis using approximate entropy and sample entropy // Am. J Physiol. Heart Circ. Physiol. 278: H2039-H2049, 2000
12. Федер Е. Фракталы / Е. Федер. – М.: Мир, 1991. – 254 с.