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ALGORITHM FOR SOLVING TWO-STAGE PROBLEM OF LOCATING PRODUCTION WITH PREFERENCES

Abstract. A two-stage task of locating production with preferences is described. The mathematical model is given. Solution algorithm based on genetic algorithm and the method of potentials is proposed. The algorithm is tested on a model problem.

Keywords: location-allocation problem, multistage location problem, genetic algorithm, optimization, location problem with preferences.

Introduction

Problems of optimal allocation of enterprises in a given area have been studied for over a hundred years. It remains the center of attention for many researchers, which results in the large number of publications devoted to this problem [1]. Such problems arise in the strategic planning of the area of interest of businesses (locating warehouses, shops, service outlets, etc.) and public institutions (schools, hospitals, fire stations, etc.), which makes these problems practical and interesting. In general, the location problem can be described as follows: it is necessary to determine the optimal location of objects according to some quality criterion that satisfies customer requirements and meets the set limits. Appropriate mathematical models can be described as a discrete and continuous formulation in [1, 2].

In this paper we consider a two-stage facility location problem. It should be noted that there are a number of areas where there are problems of this kind, since they reflect the successive processes of manufacture of one product, deliver it to the intermediate points and then deliver it to end users. In the simplest tasks of such productions two products are considered - "raw product" and "finished product ", however more product names are possible: "raw product", "semi-finished product", "finished product". One example of a two-level production process is the extraction and processing of natural resources - oil, ore, etc. Multi-stage location problem is considered in [1, 3, 4, 5].

Formulation of a problem

Problem description of multistage location problem can be formulated as follows: it is necessary to locate production including the phase 1company and phase 2 company in the area, so that the total cost of shipping raw materials and products were minimal. It is assumed that possible locations of enterprises of stages 1 and 2, as well as the location of the customers, are known in advance.

To construct a mathematical model , we introduce the following notation : Ω – an area in which you place the enterprise; N – the required amount of stage I enterprises; M_1 , M_2 – of possible locations for manufactures stages I and II respectively; K – set of consumers; c_{ij}^{I} – postage unit of raw material from the i-th stage I enterprise to j-th stage II enterprise; c_{jk}^{II} – postage from the j-th stage II enterprise to the k-th consumer; b_k – demand of k-th consumer; A^r – the cost of installing the i- th enterprise r-the stage; v_{ij}^{I} – the volume of products delivered by the i- th stage I enterprise to j- th stage II enterprise; v_{jk}^{II} – production volume delivered from the j- th stage II enterprise to the k- th consumer.

Let us assume that

$$x_i = \begin{cases} 1, \text{ if stage I enterprise is located in point } i, \\ 0, \text{otherwise.} \end{cases}$$

$$\lambda_j = \begin{cases} 1, \text{if stage II enterprise is located in point } j, \\ 0, \text{otherwise.} \end{cases}$$

Then the mathematical model can be presented as:

Minimize

$$\sum_{i \in M_1} A_i^I x_i + \sum_{j \in M_2} A_j^{II} \lambda_j + \sum_{i \in M_1} \sum_{j \in M_2} c_{ij}^I v_{ij}^I + \sum_{j \in M_2} \sum_{k \in N} c_{jk}^{II} v_{jk}^{II} \tag{1}$$

with constraints

$$\sum_{j=1}^{M} v_{ij}^I \lambda_j = b_i^I , \; i = 1,2,\dots N , \tag{2}$$

$$\sum_{i=1}^{N} v_{ij}^I = b_j^{II} , \; j = 1,2.\dots M . \tag{3}$$

$$\sum_{j=1}^{M} v_{jk}^{II} \lambda_j = b_k , \; k = 1,2,\dots K , \tag{4}$$

$$v_{ij}^I \ge 0 , \; v_{jk}^{II} \ge 0 , \; i = 1,2,...N , \; j = 1,2,...M , \; k = 1,2,...K , \tag{5}$$

$$x_i \in \{0;1\} ; \lambda_j \in \{0;1\} , \tag{6}$$

Here constraints (2) indicate that the amount of product exported from the i-th stage I enterprise must comply with the production capacity of the enterprise, constraints (3), (4) - meet the demand of stage II enterprises and consumers.

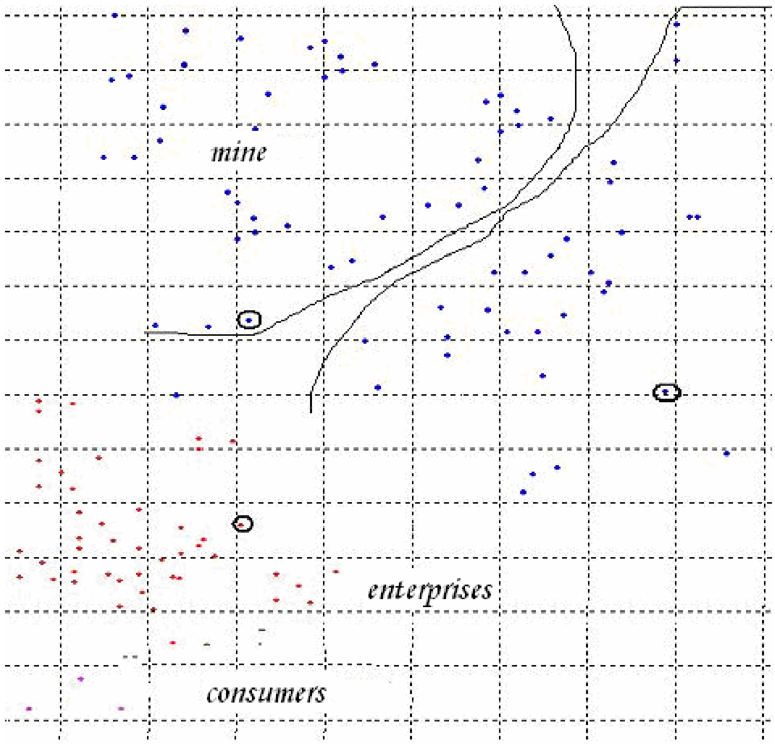

Figure 1 - Allocation of concentration plant

A similar problem in a discrete setting with two stages in the chain is considered in [4] , similar methods of solution of the tasks were also considered in [1]. [3] proposed an approach for solving the problem of optimal allocation based on the sequential solution of OPS problem [2] and the problem in (1) - (6). However, it was found that to locate stage 1 enterprises locations for facilities were selected by the shortest possible distance (cost) to stage 2 enterprises without the "quality" of these locations taken into account (Fig. 1). This result is justified if all possible sites are the same, but if their "quality" is different, then the result can not be considered acceptable. Thus, there is a need to consider the differences between the possible allocations. One approach to solving this problem is based on the use of game theory proposed by V. L. Beresnev in [6]

Main part

In this paper we propose to introduce the coefficients of "quality" β_i of possible locations to accommodate the preferences of facilities allocation. We offer to solve the problem of the form:

Minimize

$$\sum_{i\in M_1} A_i^I x_i + \sum_{j\in M_2} A_j^{II} \lambda_j + \sum_{i\in M_1}\sum_{j\in M_2} \beta_i c_{ij}^I v_{ij}^I + \sum_{j\in M_2}\sum_{k\in N} c_{jk}^{II} v_{jk}^{II}, \qquad (7)$$

with constraints (2) - (6) .

To solve this problem we propose an algorithm based on genetic algorithm and the method of potentials.

Algorithm

1. Choosing an initial population consisting of arrays with possible initial allocation of stages 1 and 2 companies.

2 .Calculation of distance from stage 1 enterprises to stage 2 enterprises, and from stage 2 enterprises to consumers.

3 .Solution of the transport problem by potential allocation for stages 1 and 2 companies.

4 .Compute the objective function by the formula (7).

5 Filling memory array with better value of the objective functional and corresponding numbers of companies.

6. Selecting the "parents", crossover, and mutation of new accommodation options.

7. Repetiting 2 - 4 for the new solutions.

8. Adding new solutions to memory array and removing the worst solution.

9. Termination check: if "yes" - go to the next step, otherwise return to step 6.

10 .Displays the better results and graphics.

Algorithm is described.

The proposed algorithm was numerically implemented and tested on the following model problem::

Suppose a consumer product is in the area $\Omega = \{(x, y) | 0 \le x \le 30,\ 10 \le y \le 30\}$. Location coordinates of consumers are known. Production is carried out in two stages. Experts identified possible locations for stage I and II companies: $M_1 = \{\tau_1^I, \tau_2^I, ..., \tau_{20}^I\}$, $M_2 = \{\tau_1^{II}, \tau_2^{II}, ..., \tau_{20}^{II}\}$ and evaluated the quality of possible locations for the stage I companies on a 10- point scale: α ={4, 8, 1, 8, 3, 3, 8, 1, 7, 9, 3, 8, 6, 2, 3, 3, 8, 10, 4, 9}, where 1 – s the least preferred location for, 10 – is most preferred. The capacities of each stage are also known: $b^I = \{50,150,150,100,50,100\}$ and $b^{II} = \{100,200,250,50\}$ and the consumer demand is $b = \{100; 100; 100; 150; 50; 100\}$. 6 stage I companies and 4 stage II companies are to be allocated taking into account the location of 6 consumers, and transported products volume is to be determined at each stage so as to minimize the functional of total costs of production and delivery to consumers:

$$F = \sum_{i=1}^{M_1}\sum_{j=1}^{M_2} \beta_i c^I{}_{ij} v^I{}_{ij} x_i + \sum_{k=1}^{K}\sum_{j=1}^{M_2} c^{II}{}_{jk} v^{II}{}_{jk} \lambda_j$$

with constraints (6) – (8).

To solve the above problem the algorithm described above has been applied. Runtime of T = 30 seconds was selected as a stopping criterion.

Table. 1 and Fig. 2 show the results of the algorithm. Asterisks indicate possible location of stage I companies (r_i) and the corresponding estimates αi, circles indicate possible locations of stage II companies (f_j), squares indicate the location of the consumers (p_k). The locations of companies are highlighted.

The results showed that the solution of two-step problem of locating gives adequate quality solution and this method of solution can be further applied.

Table 1

Numerical results of calculations using the algorithm 1 in the presence of coefficient of "quality"

stage I company			stage II company			consumers	
Coordinates of the allocation point	Traffic volume of stage I	Capaci ty	Coordinates of the allocation point	Traffic volume of stage I	Capaci ty	Coordinates of the allocation point	De mand
(6, 23)	50	50	(9, 13)	100	100	(9, 10)	100
(26, 20)	100	150	(23, 11)	50	250	(25, 10)	100
(26, 20)	50	150	(26, 11)	50	50	(25, 10)	100
(21, 25)	150	150	(17, 12)	150	200	(19, 17)	150
(27, 23)	50	100	(17, 12)	50	200	(16, 10)	50
(27, 23)	50	100	(23, 11)	100	250	(24, 1)	100
(13, 25)	50	50	(9, 13)	100	100	(9, 10)	100
(29, 21)	100	100	(23, 11)	100	250	(14, 2)	100
The objective function				10815,2			

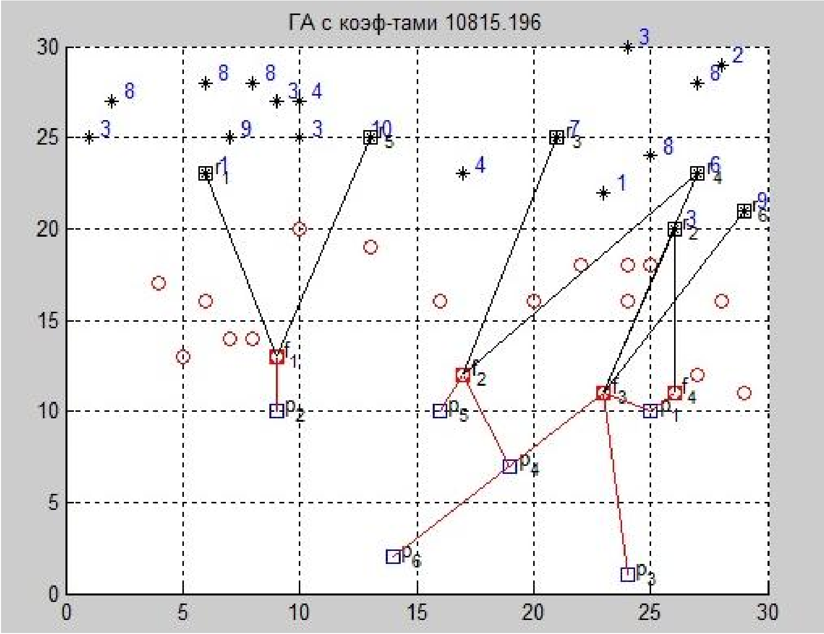

Figure 2 - The solution of the model problem with the coefficient of "quality"

For comparison, this problem was solved without quality of possible locations of stage I companies taken into consideration. Table. 2 and in Fig. 3 show the results of the algorithm under the condition of equivalence of possible locations for the first stage.

Table 2.

Numerical results of calculations using the algorithm without "quality" coefficient

stage I company			stage II company			consumers	
Coordinates of the allocation point	Traffic volume of stage I	Capacity	Coordinates of the allocation point	Traffic volume of stage I	Capacity	Координаты точки размещения	Спрос
(10, 25)	50	50	(9, 13)	100	200	(9, 10)	100
(6, 23)	150	150	(9, 13)	100	200	(14, 2)	100
(26, 20)	150	150	(24, 16)	100	250	(25, 10)	100
(29,21)	50	100	(24, 16)	50	250	(24, 1)	100
(29,21)	50	100	(28, 16)	50	50	(24, 1)	100
(23, 22)	50	50	(24, 16)	100	250	(19, 7)	150
(17, 23)	100	100	(17, 12)	50	100	(19, 7)	150
(17, 23)	100	100	(17, 12)	50	100	(16, 10)	50
The objective function			9905				

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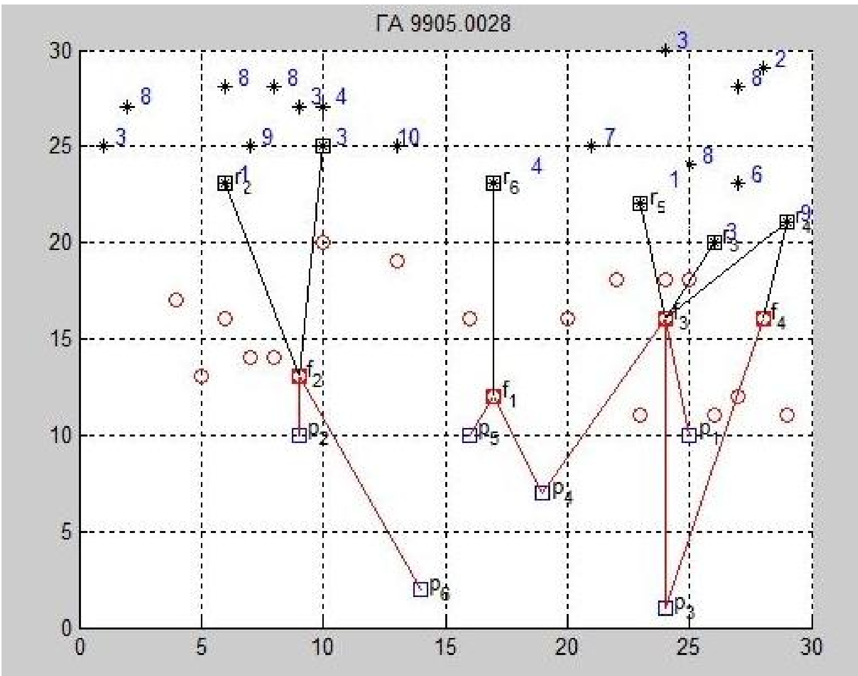

Figure 3 - The results of the calculations without the "quality" of location

As can be seen from the results of numerical experiments, the use of quality factors allows considering preferences when locating enterprises.

Conclusions

Two-stage facility location problem is relevant, because it opens the possibility for the development of new modelling techniques, innovative algorithms of solutions and interesting applications. In this paper we formulated a mathematical model of two-stage facility location-allocation problem with preferences and proposed a solution algorithm based on genetic algorithm and the method of potentials. The numerical experiments suggest the possibility of using the proposed algorithm.

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