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A.I. Guda , A.I. Mikhalyov

CHAOTIC SYSTEM WITH RELAY HYSTERESIS IN RETURN FORCE: SIMULATION AND ANALYSIS

Abstract. This article is devoted to problem of simulation of semi-harmonic dynamic system, with non-linear return force, presented by relay hysteresis. Different modes of operation are researched. Influences of system parameters are investigated.

Keywords: dynamic chaotic system, hysteresis, simulation.

Introduction

Modern control systems often have many non-linear elements as its parts, mainly to provide properties and opportunities, unavailable for linear system. But in some condition such complex systems tends to demonstrate chaotic behaviour. In some cases chaotic dynamics is treated as bad and inadmissible. In others – may provide required features. So, the problems of simulation such systems and operation mode investigation are actual.

Aims and model

Let's consider nonlinear oscillating system with relay-alike hysteresis in return force:

$$\ddot{x} + c\dot{x} + r(x,\ldots) = u(t), \qquad (1)$$

where x – coordinate, c – dumping coefficient, $r(x,...)$ – return force with hysteresis, $u(t) = U_{in}\sin(\omega_{in}t)$ – harmonic external force.

Return force is combined from two parts:

a) Linear: $r_l(x) = -ax, a > 0$. This part lead to instability is area near zero point.

b) Relay stabilization part. This part is zero near x=0, but began to limit system dynamics near $|x| \approx x_s$. Due to relay hysteresis with given with width x_0, this part activates when $x = x_s + x_0$, and deactivates when $x = x_s - x_0$. Left branch of graphical representation is symmetrical to right. Net return force represented in fig. 1.

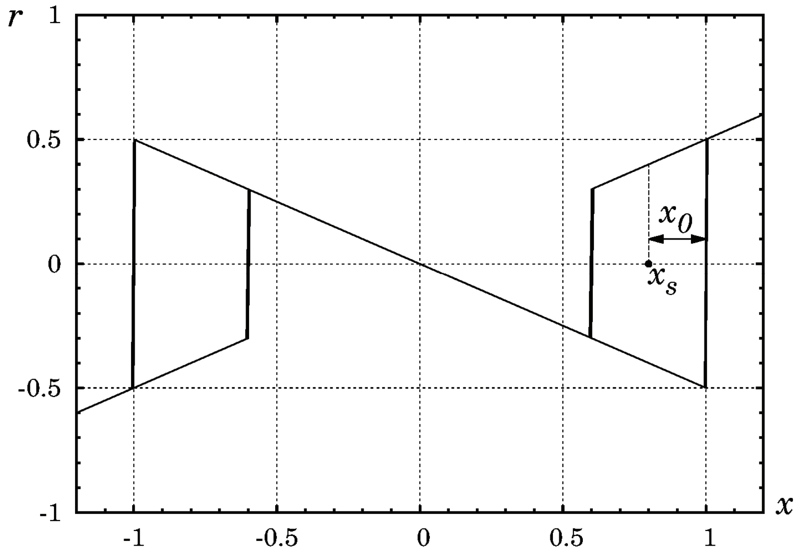

Fig 1 – Return force $r(x,...)$

Thus, system (1) demonstrates instability near x=0, but its motion is limited in general. This condition provides good background for chaotic dynamics appearance. In absence of external perturbation, system under consideration may show different kinds of self-oscillation movement. In case of harmonic external force, different modes of interaction between oscillations will occur, and may lead to chaotic behaviour.

So, the main aim of this paper is to simulate the dynamic of given system (1) in different conditions, and to determine, is chaotic modes occurs. In this case, the aim is to investigate and discover values of parameters, which result to such behaviour.

Simulation

Simulation of dynamic system (1) was done by the means of developed computer simulation program "qmo2". This program appointed to simulation of different kins of dynamic systems. The main difference between this program and analogs is specialization on essential nonlinear systems. The view of the simulated system with example of result plot is shown in fig. 2.

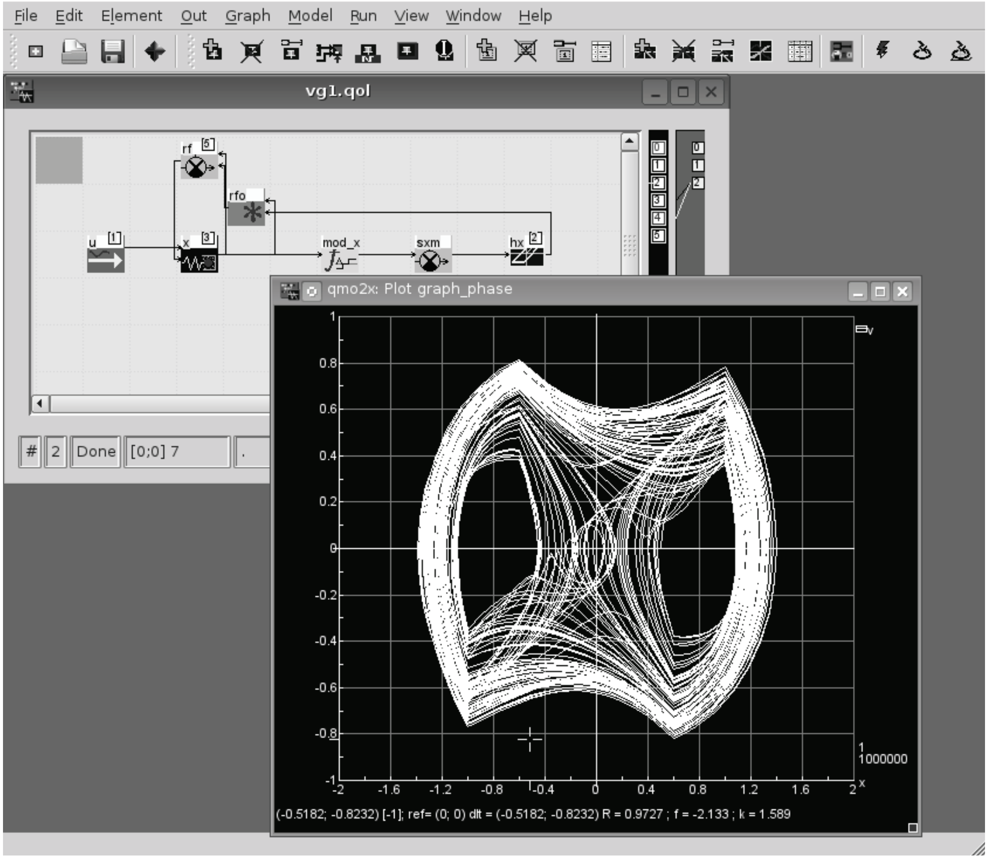

Fig.2 – Simulation of system (1) in “qmo2” program

First of all, the simulation in case of zero external force was done. Some results (phase portrait and spectrum) are show in fig. 3.

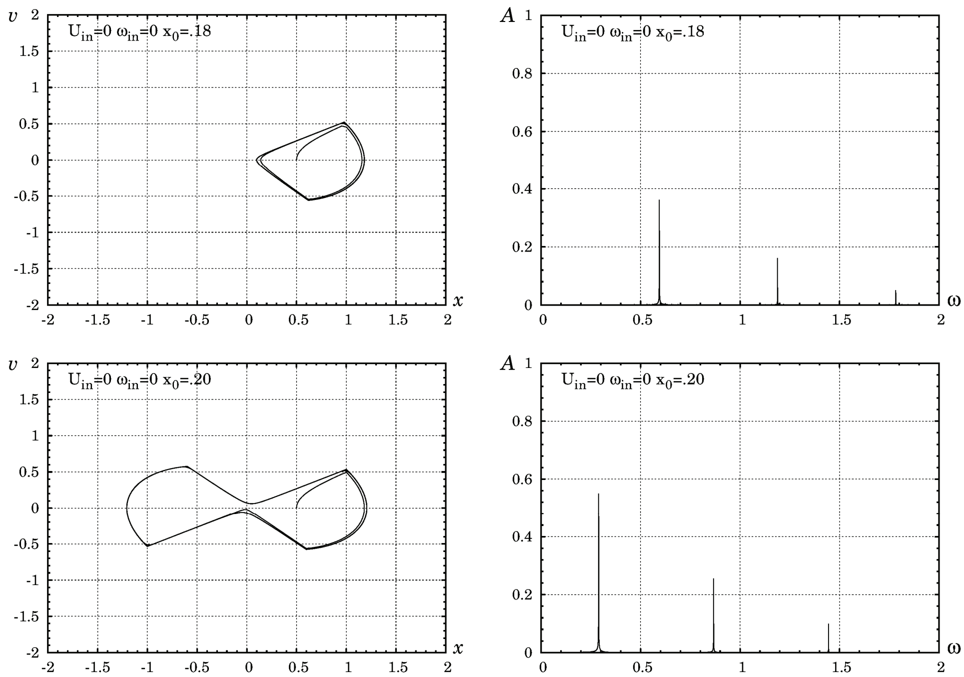

Fig. 3 – System dynamic in case of $U_{in} = 0$

These results shows, that small alteration of system parameters results to essential realignment of trajectories. Nevertheless, the dynamic is quite simple, which confirmed with pure spectrum, with peaks on 3, 5, 7 order of base frequency.

Then series of simulation was held with fixed amplitude and frequency of input signal $u(t)$. The values of ω_{in} was chosen quite far apart of system base frequency. The value of parameter x_0 – with of hysteresis was altered in area $(0, x_s)$. Most prominent results are show on fig. 4.

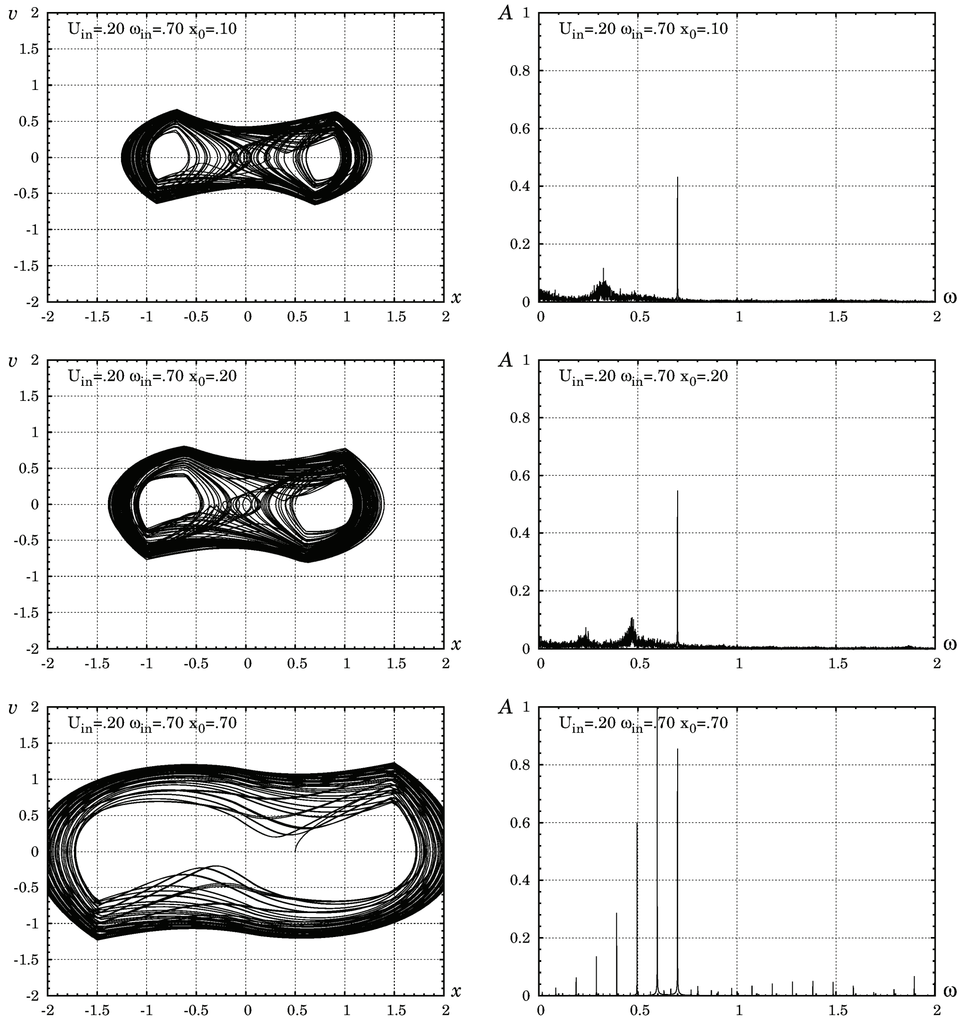

Fig. 4 – System dynamic in case of $U_{in} = 0.2, \omega_{in} = 0.7$

First of all, it worth to mention, that in case of even small input signal two-looped phase portrait is appears on lesser values of x_0. Moreover, the system response in ω_{in} is increasing while x_0 raised. It may be due to fact, that value of x_0 determines the square of loop in fig. 5.

This, in order, defined the amount of energy, what system receives at one oscillation.

The complexity of phase portraits and contiguous spectrum is a good indicator of chaotic dynamic. While x_0 raises to maximum (x_s), the influence of the input signal became more negligible, and system demonstrates complex-periodic movement.

Next series of simulation was held on fixed x_0, ω_{in}, and altering U_{in} (fig. 5).

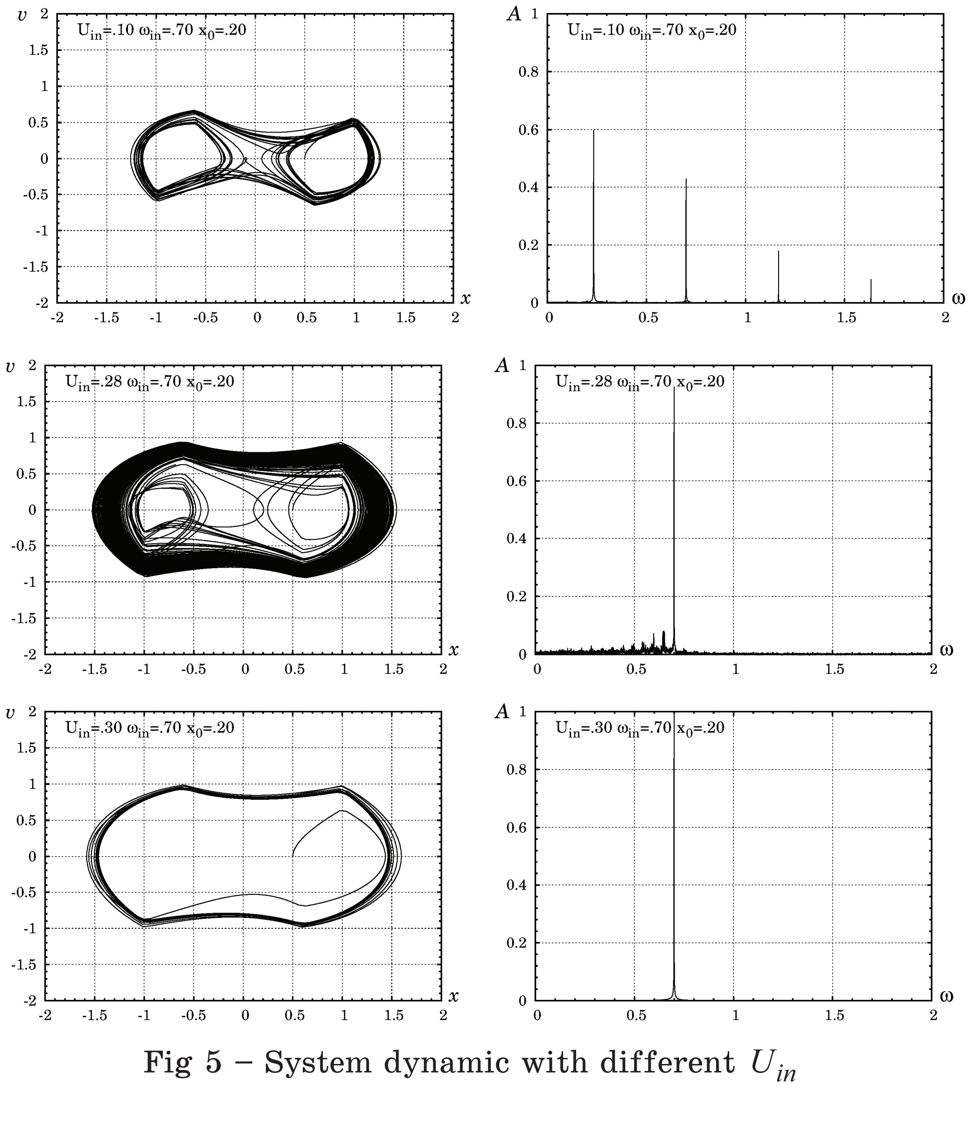

Fig 5 – System dynamic with different U_{in}

As a predicted, it exist a quite small range of U_{in}, in which system displays chaotic dynamic. At small values of U_{in} the influence of input signal is negligible. Otherwise, at large values of U_{in}, input signal fully quells this inner system dynamic.

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The last series of simulation was held with fixed x_0, U_{in}, and variation of ω_{in} was done.

It was found, that in this case multiple alternations of system mode occurs. The system behaviour changes from complex-periodic to chaotic and vice versa. In case of “frequency capture” inner and external oscillations became synchronous, which lead to pure spectrum and simple phase portrait. In different condition chaotic movement was appears.

In comparison with well-known Van-der-Pole system, system under consideration show chaotic behaviour in much wide ranges of parameters. For example, all simulation shown above results was simulated with value of damping coefficient $c = 0.4$. This value is large enough to suppress complex dynamic of many systems. Such wide range of chaotic mode in system (1) is conditioned by essential non-linearity of return force.

Conclusions

Results of simulation allow us to make some conclusions:

- system (1) demonstrates both complex-periodic and chaotic dynamic;
- the most order of influence to dynamic belong to parameters x_0 and ω_{in};
- due to essential non-linearity of return force the ranges of parameters values, in which chaotic mode is observed, is comparatively large.

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