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# MATHEMATICAL MODEL OF TRAFFIC ROUTING IN A HETEROGENEOUS INFORMATION ENVIRONMENT

Annotation. A mathematical model for finding the optimal route and its value in the presence of diverse routing requirements in a heterogeneous information and communication network of any unstable structures, which allows the most efficient use of network resources by reducing the number of locks and traffic delays.

Keywords: routing, traffic management, resource planning, information communications management, distributed heterogeneous networks.

## Introduction

Any distributed information system primarily involves a network of nodes. Therefore, one of the most important tasks in the distributed network is routing. There is currently a large number of routing algorithms, satisfying the requirements for the transfer of traffic, for quality of service parameters, service level agreements, etc. [1-5]. Thus, practically all the algorithms are designed for stable network and do not consider the mobile nodes and the heterogeneity of the structure. In this article we consider a mathematical model for the routing of modern heterogeneous information and communication networks with unstable structure.

All values specified in this article are the normalized (conditional) and they can be used for different dimensions and conditions: data throughput and traffic volume are measured in units convenient for a specific task (bits, bytes, packets, etc.), the cost of transmission defines a certain indicator for the transfer costs: transmission time, energy or the economic costs of transmission, etc.

## Mathematical model of routing

We consider a distributed information system (DIS) and the corresponding complete oriented graph $G = (V, E)$, where $V$ – a plurality of nodes, $E$ – a plurality of communication lines (routes) between each pair of nodes.

We define the plurality $R$, that $R \subset V$ x $V$. Pairs of nodes from the plurality $R$ correspond to pairs of end nodes of the network between which the traffic is transferred.

For $\forall(v_i, v_j) \in R$ we define the plurality of all possible routes $L_{ij} =\{l_1(v_i, v_j), l_2(v_i, v_j), \ldots ,l_n(v_i, v_j)\}$ between nodes $(v_i, v_j)$, where $l_r(v_i, v_j)$ – some unique route between nodes $(v_i, v_j)$.

In the work [6] were identified for each pair of nodes $(v_i, v_j) \in R$ function $f(v_i, v_j) \geq 0$, describing the volume of traffic between these nodes, and the conditions for such a function, excluding the weight fractions of the bandwidth of each route. We determine the conditions that satisfy the function $f(v_i, v_j)$ taking into account the weight fractions of bandwidth of the route $w$:

$$F(p, v_i, v_j) = \sum_k f_{ij}(p, v_i, v_k) = \sum_k f_{ij}(p, v_k, v_j); \quad \forall(v_k) \in V \setminus \{v_i, v_j\}; \tag{1}$$

$$F_{ij}(p, v_k, v_l) \leq \frac{w_{kl}}{w_{ij}} f(p, v_i, v_j) - \sum_{m,n} F_{mn}(p, v_k, v_l); \quad (v_m, v_n) \in R \setminus (v_i, v_j), \tag{2}$$

where $F_{ij}(v_k ,v_l)$ – traffic proportion $F(v_i, v_i)$, proceeding between nodes $(v_k ,v_l)$; $w_{kl} \geq 0$ – bandwidth of the route between nodes $(v_k ,v_l)$, $p$ – data flow identifier.

Condition (1) specifies that the volume of traffic transferred over networks from the node $v_i$, will be equal to the amount of traffic coming into the node $v_j$. Condition (2) means that the volume of traffic transferred over any route, does not exceed the bandwidth of this route.

We consider DIS as a graph, each pair of nodes $(v_i, v_j)$ and the route between them $(v_i, v_j)$ for which is assigned the tuple:

$$[w_{ij}, prob_{ij}, L_{ij}, f(v_i, v_j, t)], \tag{3}$$

where $prob_{ij}$ – the probability value of the existence of at least one route between nodes $(v_i, v_j)$, $f(vi, vj, t) \geq 0$ – function corresponding to the total amount of traffic being transferred between nodes $(v_i, v_j)$ at each time point $t$:

$$f(p, v_i, v_j, t) = \frac{dF(p, v_i, v_j)}{dt} \tag{4}$$

To each individual route $l_r \in L_{ij}$ corresponds the tuple:

$$\begin{gathered}[w_r(v_i, v_j), prob_r(v_i, v_j), \cos t_r(v_i, v_j), f_r(p, v_i, v_j, t)];\\ 0 \leq prob_r(v_i, v_j) \leq 1;\\ 0 \leq \cos t_r(v_i, v_j),\end{gathered} \tag{5}$$

where $prob_r(v_i, v_j)$ – the probability value of the existence of the route $l_r$ between nodes $(v_i, v_j)$, $cost_r(v_i, v_j)$ – the value of the transfer cost of any conventional information unit on the route $l_r$ between nodes $(v_i, v_j)$, $f_{r(}(p, v_i, v_j, t) \geq 0$ – function corresponding to the amount of traffic being transferred on the route $l_r$ at each time point $t$. At the same time we assume that the value of $prob_{ij}$, $prob_r(v_i, v_j)$ and $cost_r(v_i, v_j)$ are the same for the traffic transfer in both directions (i.e. the probability of existence of the route and the transfer cost on this route in the forward direction are equal to the probability of the existence of this route and transfer cost on this route in the opposite direction).

We assume that the probability of existence of the route between two nodes of each unique route $l_r$ is the probability product of all intermediate routes (included in this route) between adjacent links on this route:

$$\begin{gathered}prob_r(v_i, v_j) = \prod_{m,n} prob_r(v_m, v_n);\\ \forall l_r(v_m, v_n) \in l_r(v_i, v_j),\end{gathered} \tag{6}$$

where $r$ – conditional number of unique route between each pair of nodes $(v_i, v_j)$. Here and after we will take into account only routes with a probability different from zero, as the route with a probability of zero will never be able to realize itself.

We assume that the overall probability of the existence of at least one route between a pair of nodes $(v_i, v_j)$ is the sum of probabilities of the existence of unique routes between the nodes:

$$prob(v_i, v_j) = 1 - \prod_{\forall r} \left(1 - prob_r(v_i, v_j)\right); \quad \forall l_r(v_i, v_j) \in L_{ij}, \tag{7.1}$$

$$prob(v_i, v_j) = 1 - \prod_{\forall r} \left(1 - \prod_{m,n} prob_r(v_m, v_n)\right); \quad \forall l_r(v_m, v_n) \in l_r(v_i, v_j) \in L_{ij} \tag{7.2}$$

The formula (7.2) enables us to calculate the the probability of existence of at least one route between two heterogeneous network nodes, based on the probability of the existence of each individual networks segment.

The remaining bandwidth portion of the route $l_r$ between the intermediate nodes $(v_m, v_n)$ – is the value showing on what value can be increased the flow of data $p^*$ on this route after the deduction of all third-party data flows:

$$w_r^*(p^*, v_m, v_n, t) = w_r(v_m, v_n) - \sum_{\forall g \in G} f_r(p, v_m, v_n, t), \forall p \in P \setminus p^* \tag{8.1}$$

where $P$ – plurality of data flows coming on the route $l_r$. The remaining bandwidth portion of the route $l_r$ between the intermediate nodes $(v_i, v_j)$ will correspond to a segment of the route with the lowest bandwidth:

$$w_r^*(p^*, v_i, v_j, t) = \min_{\forall(v_m, v_n)} \left(w_r^*(p^*, v_m, v_n, t)\right); \quad \forall(v_m, v_n) \in (v_m, v_n) \tag{8.2}$$

The amount of information transmitted in the flow $p^*$ on the route $l_r$ between a pair of nodes $(v_i, v_j)$ during the time T:

$$F_r(p^*, v_i, v_j) \leq \int_0^T w_r^*(p^*, v_i, v_j, t)\,dt \tag{9}$$

The value of a unit of information transfer for a pair of nodes $(v_i, v_j)$ on each unique route $l_r$ will be assumes as the sum of the values of the data transfer on each fragment of this route:

$$cost_r(v_i, v_j) = \sum_{m,n} cost_r(v_m, v_n) \frac{F_r(p^*, v_m, v_n)}{F_r(p^*, v_i, v_j)} \tag{10}$$

The relative cost of traffic transfer for each unique route $l_r$ will be assumes as the ratio of the cost of the information transfer on this route to the probability of the existence of this route:

$$cost_r^*\left(v_i,v_j\right)=\frac{cost_r(v_i,v_j)}{prob_r(v_i,v_j)};$$
$$prob_r\left(v_i,v_j\right)>0 \qquad (11)$$

It is obvious that the route with the lowest relative value will be the the most optimal route of data transfer: such a route may have a higher value than a route with minimum costs, but the probability of losses on such a route will be much lower:

$$cost_o\left(v_i,v_j\right)=\min_r\left(cost_r^o\left(v_i,v_j\right)\right)=\min_r\left(\frac{cost_r\left(v_i,v_j\right)}{prob_r\left(v_i,v_j\right)}\right);$$
$$prob_r\left(v_i,v_j\right)>0, \qquad (12)$$

where $cost_o$ – the costs of data transfer between nodes $(v_i, v_j)$ on the most optimal route.

## Conclusions

We solve the problem of finding the optimal route for traffic transfer taking into account the network load, remaining bandwidth of its links and routing requirements on condition of an unstable structure of the network, traffic transfer costs and the possibility of its separation.

The suggested mathematical model can be used to develop methods and algorithms for routing, finding a solution for the problem of routing in information and communication networks with a complex structure. The implementation of the model is carried out by splitting the network graph on components, for each of them is applied the set out routing model followed by the composition of the total solution.

## REFERENCES

1. Capone A., Fratta L., Martignon F. Dynamic online QoS routing schemes: performance and bounds // Computer Networks. 2006. No. 50. P. 966-981.

2. Capone A., Fratta L., Martignon F. Dynamic routing of bandwidth guaranteed connections in MPLS networks // Intern. Journal on Wireless & Optical Communications. 2003. No. 1. pp. 75-86

3. Kwong Kin-Wah, Guerin R., Shakh A., Shu Tao. Balancing performance, robustness and flexibility in routing systems // Proc. of ACM CoNEXT. Madrid, 2008. pp. 372-383.

4. Hadjiona M., Georgiou C., Papa M., Vassiliou V. A hybrid fault-tolerant algorithm for MPLS networks // Proc. of WWIC. Tampere. 2008. pp. 41-52.

5. Efremov A.S., Zelencov V.A., Mironov, A.S., Ulasen S.N. Forecasting of a residual resource of communication equipment // Herald of communication. 2004. No. 4. pp. 93-99.

6. Zelencov V.A., Tsivirko E.G., Chukarin A.V. The traffic routing method in the information and communication heterogeneous network // News of Higher Schools. 2010. No. 11. Book 53. pp. 56-61.

7. Berezko M.P., Vishnevsky V.M., Lewner E.V., Fedotov E.V. Mathematical models of the study the routing algorithms in data transfer networks // Information processes. 2001. Book. 1, No. 2. pp. 103-125.

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Купін А. І., Піддубний Б. А., Музика І. О. Математична модель маршрутизації трафіку в гетерогенному інформаційному середовищі // Системні технології. Регіональний міжвузівський збірник наукових праць. - Випуск ? (??). - Дніпро, 2017. - С. ?? - ??.

Наведено математичну модель пошуку оптимального маршруту і його вартості при наявності різнорідних вимог до маршрутизації в гетерогенної інформаційно-комунікаційної мережі довільної нестабільної структури, що дозволяє максимально ефективно використовувати ресурси мережі, зменшуючи кількість блокувань і затримок трафіку.

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Купин А. И., Поддубный Б. А., Музыка И. О. Математическая модель маршрутизации трафика в гетерогенной информационной среде // Системные технологии. Региональный межвузовский сборник научных трудов. - Выпуск ? (??). - Днепр, 2017. - С. ?? - ??.

Приведена математическая модель поиска оптимального маршрута и его стоимости при наличии разнородных требований к маршрутизации в гетерогенной информационно-коммуникационной сети произвольной нестабильной структуры, позволяющий максимально эффективно использовать ресурсы сети, уменьшая количество блокировок и задержек трафика.

Библ. 7.

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Mathematical model of routing in heterogeneous distributed information system

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The article provides a mathematical model of finding the optimal route with minimal cost in the presence of heterogeneous requirements for routing in the distributed information communication network, which allows the most efficient use of network resources by reducing the number of locks and traffic delays. Theoretical research is confirmed by modeling of traffic routing on the example of a fragment of a heterogeneous network.

Bibl. 7.