UDC 531.314.2+536.242+532.516.5+519.683.2

K.S. Krasnikov, S.E. Samokhvalov, V.P. Piptyuk

THE MATHEMATICAL MODEL OF THE CORED WIRE INJECTION IN THE MOLTEN STEEL DURING BLOWING ON THE LADLE-FURNACE

Abstract. The mathematical model of the industrial process is presented. Three-dimensional movement of the wire, one-dimensional melting of the wire with a crust and three-dimensional movement of the molten steel with powder averaging in it is taken into account. The mathematical model is implemented in the computer program to check its quality and to make computational experiments.

Keywords: nonlinear system, three-dimensional space, computer model, cored wire, powder, molten steel, furnace-ladle.

Introduction

Cored wire injection is a modern method of molten steel refining. Finding of rational technological parameters of this process is expensive in industrial and laboratory experiments. This paper presents a mathematical model for numerical experiments on the computer, which are cheaper and widely used today.

Related work

In work [1] the authors considered the thermodynamics of wire in the cross-section with the crust of slag and steel. In [2] the authors neglected appearance of the crust. We think that thermodynamics is not enough to reproduce the process of wire injection, because in addition to wire melting there are three-dimensional motion of the wire and powder averaging affected by hydrodynamics during gas blowing. In [3] a three-dimensional mathematical model of the wire motion was implemented in the computer program and the adequacy of the model was tested, but ability of free movement of wire`s part was not taken into account. In [4] a three-dimensional model of granule melting with crust was created – we will use it for one-dimensional wire melting. In [5] suitable to our problem three-dimensional model of the hydrodynamics was constructed in cylindrical coordinates and the results of numerical experiments were

presented. The authors of the work [6] offered own method of cored wire input, in which the parts of the wire can disconnect – in this article we will take the free movement of these parts into account.

Mathematical model

The article aims to build a model of cored wire injection in the melt during blowing on ladle-furnace. Given the complexity of model we will build it by solving the following tasks:

1) The problem of three-dimensional motion of cored wire, as a system of solids. A three-dimensional solution is needed to determine the trajectory and coordinates of powder`s or wire`s part release;

2) One-dimensional problem of wire melting with crust appearance. Given the small thickness of the wire shell (~ 1 mm), heat flow along the wire is neglected;

3) The problem of three-dimensional motion of the molten steel and powder averaging in it. Different locations and number of blowing tuyeres cause the finding of three-dimensional velocity field of the melt.

Let's go to first problem. Cored wire is modeled by chain of rods. The force of hydrodynamic resistance of the melt, the force of gravity, the buoyancy and force of bending elasticity act on the rods. Schematically chain of rods is shown in Figure 1. Reference point is a ladle bottom center.

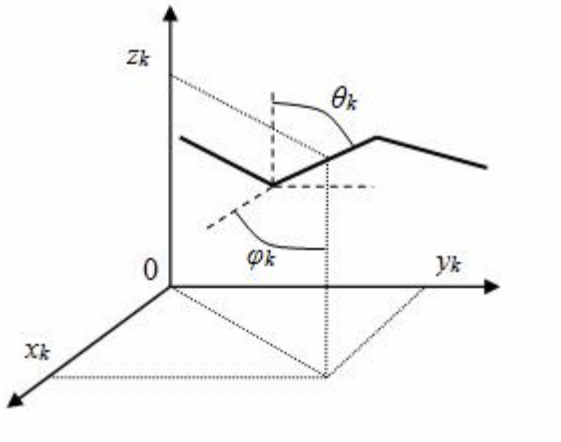

Fig. 1. Cartesian and spherical coordinates of k-th rod

Assumptions:

1) The wire is unstretchable and incompressible;

2) The torsion of the wire is neglected;

3) The angles between the rods are small, so Hooke's law performs when wire bends;

4) The melt`s resistance to rod rotation is neglected and only the component of velocity, which is perpendicular to rod, is taken into account to determine the resistance;

The dynamics of the rod`s system is determined by Lagrange equations of the second kind [3, 7, 8], taking into account the potential and dissipative forces:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = -\frac{\partial U}{\partial q_i} + \tilde{Q}_i \quad i = \overline{1,S} \tag{1}$$

where T and U – respectively the kinetic and potential energy of the rod system, $\widetilde{Q}_i$ – generalized dissipative forces, q_i – generalized coordinates. We choose angles of a spherical coordinate system as generalized coordinates: polar θ and azimuthal φ. Number of freedom degrees S is dual count of rods.

The kinetic energy T (1) is determined using Cartesian coordinates of the centers of rods masses and their velocities expressed in generalized coordinates θ and φ:

$$x_k = x_0 + l\sum_{j=1}^{k-1}\sin\theta_j\cos\phi_j + \frac{l}{2}\sin\theta_k\cos\phi_k,$$

$$y_k = y_0 + l\sum_{j=1}^{k-1}\sin\theta_j\sin\phi_j + \frac{l}{2}\sin\theta_k\sin\phi_k, \tag{2}$$

$$z_k = z_0 + l\sum_{j=1}^{k-1}\cos\theta_j + \frac{l}{2}\cos\theta_k,$$

$$v_k^x = v_0^x + l\sum_{j=1}^{k-1}(\omega_j\cos\theta_j\cos\varphi_j - \sigma_j\sin\theta_j\sin\varphi_j) + \frac{l}{2}(\omega_k\cos\theta_k\cos\varphi_k - \sigma_k\sin\theta_k\sin\varphi_k),$$

$$v_k^y = v_0^y + l\sum_{j=1}^{k-1}(\omega_j\cos\theta_j\sin\varphi_j + \sigma_j\sin\theta_j\cos\varphi_j) + \frac{l}{2}(\omega_k\cos\theta_k\sin\varphi_k + \sigma_k\sin\theta_k\cos\varphi_k),$$

$$v_k^z = v_0^z - l\sum_{j=1}^{k-1}\omega_j\sin\theta_j - \frac{l}{2}\omega_k\sin\theta_k,$$

$$T = \sum_{k=1}^{N}\left[\frac{m_k}{2}(v_k^{x^2} + v_k^{y^2} + v_k^{z^2}) + l^2\frac{m_k}{24}(\omega_k^2 + \sigma_k^2\sin^2\theta_k)\right],$$

 ISSN 1562-9945

where ω and σ – angular velocities for the angles θ and φ respectively, mk – weight of k-th rod, l – length of the rod.

The potential energy of the system (1):

$$U = \sum_{k=1}^{N}\left[g(1-\frac{\rho_0}{\rho_k})m_k z_k + \frac{\kappa_k}{2}(\Delta_k - \psi_k)^2 \right],$$

where g – gravitational acceleration, ρ0 and ρ_k – respectively melt density and average density of the k-th rod (ρ0 depends on the rod location – steel, slag, air), κ_k – coefficient of elasticity in the compound of (k-1)-th and k-th rods, Δ_k – angles between them, ψ_k – angles of permanent deformation. Assuming that the angles Δ_k are small, we define them using the formula:

$$\Delta_k^2 = \Delta\theta_k^2 + \Delta\varphi_k^2 \sin^2\theta_k,$$

where $\Delta\theta_k = \theta_k - \theta_{k-1}$, $\Delta\varphi_k = \varphi_k - \varphi_{k-1}$.

Let`s find the generalized forces of resistance $\tilde{Q}_i$ (1):

$$\tilde{Q}_i = \sum_{k=1}^{N} \vec{f}_k \cdot \frac{\partial \vec{r}_k}{\partial q_i},$$

where $\vec{r}_k$ – the radius-vector of the center of k-th rod mass with coordinates x_k, y_k, z_k. Let $\vec{w}_k$ be the velocity of the mass center of the k-th rod relative to the melt, and $\vec{l}_k$ – the direction of the rod, then let`s define the resistance and rod`s perpendicular velocity component:

$$\vec{f}_k = -C_x \frac{\rho w_k^{\perp} \vec{w}_k^{\perp}}{2} A,$$
$$\vec{w}_k^{\perp} = \vec{w}_k - (\vec{l}_k \cdot \vec{w}_k)\vec{l}_k / l^2,$$

where C_x – dimensionless drag coefficient, ρ – density of the melt, A – area of longitudinal section of the rod (rod`s diameter multiplied by rod`s length).

The system (1) is not solvable analytically after differentiating, therefore, to get the numerical solution let`s divide the time axis into layers and replace derivatives by corresponding velocities and accelerations, suggesting their constancy at each time step. We have obtained a system of linear algebraic equations in which the number of unknown angular accelerations equals the number of equations. We use classical iterative Gauss-Seidel method to solve the system. The rod`s angles and

speeds for the next time step is determined from the Euler-Cromer method:

$$\omega_k^{n+1} = \omega_k^n + \alpha_k^n \Delta t,$$
$$\sigma_k^{n+1} = \sigma_k^n + \beta_k^n \Delta t,$$
$$\theta_k^{n+1} = \theta_k^n + \omega_k^{n+1} \Delta t,$$
$$\phi_k^{n+1} = \phi_k^n + \sigma_k^{n+1} \Delta t,$$

where (n+1) – the next time step, α and β – angle accelerations for θ and φ respectively, Δt - time step.

The above solution of the problem takes into account the change in the current number of rods. The length of the wire`s part in the melt increases with constant injection speed v0. The extending of this part is doing by the addition of the rod at the wire`s entry point in the time step, which is defined as the ratio of the rod length l and input speed v0.

Let`s pay attention to the components x0, y0, z0 in (2), and let`s state that they depend on the input angles and known input wire speed v0, so they are easy to identify in time. But if part of wire has been disconnected and moves independently in the melt [6], then the components x0, y0, z0 in (2) is unknown. For this case we have new three generalized coordinates (x0, y0, z0) in the mathematical model and after differentiation of equations (1) we obtain a new system with increased number of equations by three and unknowns ax0, ay0, az0. So in this way the free movement of the disconnected part has been considered.

Space for the wire`s motion is restricted by bottom and walls of the ladle. Let`s assume that taking into account the coordinates of rod mass center is sufficient for collision detection, so let`s set impact conditions of the rod with a ladle and determine the angular velocity, which recovered after collision:

$$x_k^2 + y_k^2 > R^2 \rightarrow \left[\begin{array}{l} \omega_k = \dfrac{b}{l}(v_k^x \cos\theta_k \cos\varphi_k - v_k^y \cos\theta_k \sin\varphi_k) \\ \sigma_k = \dfrac{b}{l}(v_k^x \sin\theta_k \sin\varphi_k - v_k^y \sin\theta_k \cos\varphi_k) \end{array} \right],$$

$$z_k < 0 \rightarrow \omega_k = \frac{b}{l} v_k^z \sin\theta_k,$$

where R – ladle radius, b – the coefficient of restitution of angular velocity.

 ISSN 1562-9945

Thus three-dimensional model of the wire`s motion was built. Now we turn to the problem of melting.

The mathematical model of rod`s melting is based on the finite-volume method in cylindrical coordinate system [4]. We introduce the following assumptions:

1) The heat flow along the wire is neglected and only radial flow is considered;

2) The break of crust under internal pressure and possible removing of the crust`s pieces are ignored;

3) The movement of substance inside the wire is neglected;

4) We assume that the liquid material of the wire is mixed with molten steel instantly.

Let`s assume that areas with homogeneous material occur in wire during melting. All areas consist of finite volumes. We will determine the flow of heat on the faces of volumes. Let`s consider these areas (Fig. 2):

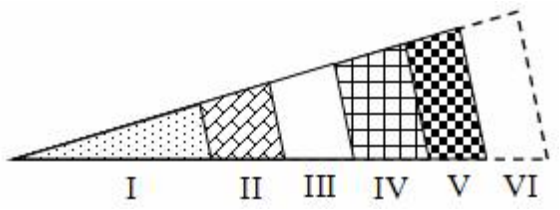

Fig. 2. Areas in schematic cross section of the cored wire (I – powder, II – wire`s shell, III – shell`s liquid material, IV – crust of melt, V – crust formation, VI – molten slag or steel)

I. Area of powder that has own thermal properties (eg low thermal conductivity);

II. Area of solid wire shell (usually thin);

III. Area of liquid material of the shell. This area occurs when shell material becomes liquid – heated above T_L^S (the liquidus temperature of shell`s material).

IV. Area of melt`s crust expands layer by layer when cold (with respect to melt) surface of the wire causes rapid heat loss in a thin layer of on-surface melt and then solidification at a temperature below T_S^M (the solidus temperature of the melt).

V. Area of crust formation has a thickness in one finite volume and is always on the surface of the wire.

VI. Ambient environment of the wire. It may be the atmosphere if the wire lies on the slag. This may be molten slag, steel or their interface, depending on the location of the wire in a given time. Thermo- and hydrodynamics is taken into account by the empirical heat transfer coefficient.

Next we will define heat flows. At the edge of areas V and VI we have convective boundary condition (III type):

$$q = \alpha \cdot S \cdot (T - T_p),$$

where a – the heat transfer coefficient, S – the area of volume face, T_p – the temperature of the melt, T – temperature in surface volume.

At the edge of areas IV and V it is a diffusion heat flow, as measured by Fourier law:

$$q_i = \lambda \cdot S \cdot (T_{i-1} - T_i) / L,$$

where λ – thermal conductivity, which is determined by the harmonic mean of corresponding thermal conductivities of neighboring cells [9], S – the area of volume face, L – distance between the centers of volumes, T_i and T_{i-1} – temperatures of corresponding volumes. Also, the law determines the thermal diffusion flows inside wire.

The heat of phase transition of the volume material included in the effective heat capacity [10].

Over time, temperature of volume material reaches the melting point of the crust and melt fills corresponding volumes. However, some volumes can have the remaining solid material inside and crust can appear again.

After shell`s melting the powder enters the steel and increases the concentration in the appropriate place of the ladle. Corresponding rod is removed. If rod has been melted in the middle of the chain, then we disconnect part of the chain and its calculation takes the free movement into account.

Thus one-dimensional problem of wire`s melting is solved with crust appearing. The advantage of the finite-volume method is conservative difference scheme. Next let`s consider hydrodynamics.

We will solve the problem of molten steel motion and powder averaging using known Navier-Stokes equations and convection-diffusion equation with the following assumptions [5]:

1) The molten steel is a homogeneous viscous incompressible Newtonian fluid;

2) The ladle has the shape of a cylinder;

3) The melt surface is smooth and only the dynamic pressure is considered;

4) The influence of the powder`s and wire`s movement to the melt hydrodynamics is neglected;

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla) \cdot \vec{v} + \nu \Delta \vec{v} - \frac{1}{\rho} \nabla p,$$
$$\nabla \cdot \vec{v} = 0,$$
$$\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (\vec{v} c),$$

where $\vec{v}$ – velocity vector field, ∇ – Del operator, Δ – vector Laplace operator, t – time, ν – kinematic viscosity coefficient, ρ – density, p – pressure field, c – concentration field of powder, D – diffusion coefficient.

On the boundaries of calculation area we set no-flow and partial slip conditions. Also, these conditions can be set inside ladle. We are solving above equations numerically in cylindrical coordinates by finite difference method in following steps:

1) The computation of the velocity field (without pressure field);

2) The calculation of the pressure field using the divergence of velocity field;

3) The projection step corrects the velocity field using the pressure field (satisfying the continuity equation);

4) The calculation of the powder concentration field using the new velocity field.

The computation of these steps was done using the explicit scheme in time. The condition for the ending of simulation is reaching 2% in coefficient of powder concentration variation.

So we have solved three problems. Each task is doing its computation simultaneously in time. The possibility of different time step values for them is taken into account. Fig. 3 shows the exchange of results between tasks at each time layer.

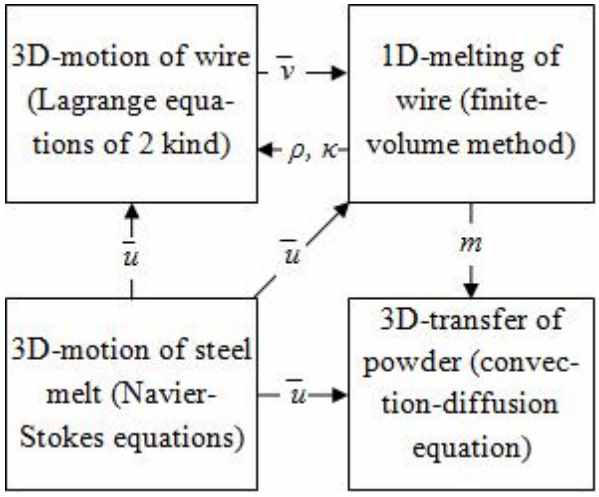

Fig. 3. The system of problems and relations between them:
$\vec{v}$ – rod velocity, κ – coefficient of elasticity in connecting rods,
ρ – density of the rod, $\vec{u}$ – melt velocity field,
m – mass of powder that is released from the wire

The computer program “Primat” was created for the numerical experiments. General flowchart of algorithm is shown in Fig. 4. Apart shown in figure the user interface allows the calculation pausing, to change the viewpoint or color of 3D-field. During the calculation the program displays the current state of the model, namely shows cored wire, sections of three-dimensional velocity field and powder, minimum and maximum powder concentration in the ladle. So, getting all the information about the state of the model, it is convenient to check. For example, hydrodynamics can be checked by the following indicator: the powder mass in the melt should remain constant over time for a given order of accuracy (starting calculation with predefined mass in melt).

Conclusions

Thus, problems of nonlinear dynamics of solids in Lagrange variables and nonlinear dynamics in continuum Euler variables are solved numerically. Also the one-dimensional problem of cylinder melting with crust appearing is solved. Usage of spherical coordinate system for cored wire implicitly considers reaction of rod to rod connections, which guarantees the continuity of the rod`s chain. Hydrodynamics model uses a cylindrical coordinate system, which naturally matches the shape of the ladle. Three-step scheme for velocity field computation was used to satisfy the continuity equation.

The mathematical model reproduces the industrial process starting from cored wire injection under the level of the melt and finishing by powder averaging during inert gas blowing. The computer program allows to make experiments, to observe the model state. Computational part of the program can be used on the high-performance computers. The mathematical model is the basis for further improvement and incorporation of neglected phenomena.

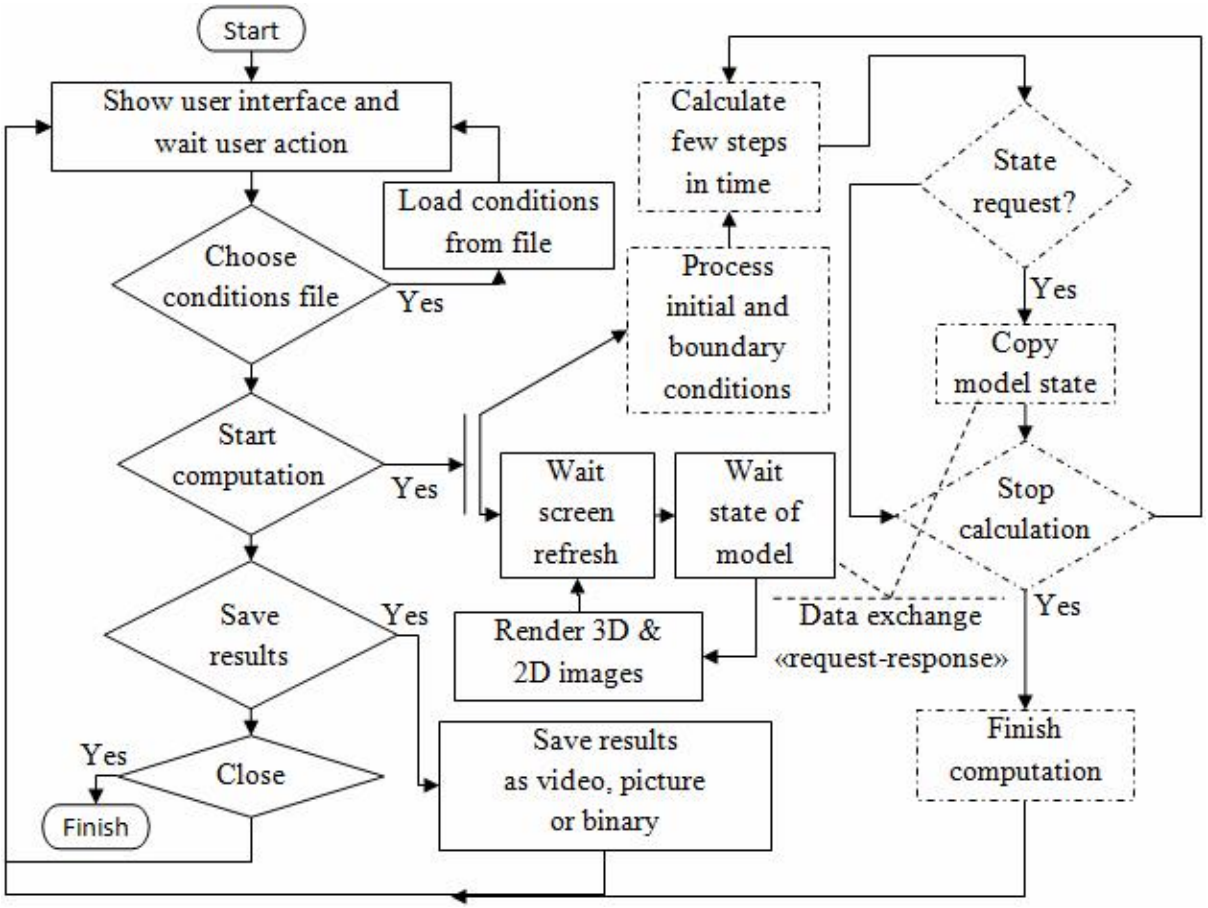

Fig. 4. General flowchart of the computer program “Primat”. Dash-marked blocks is computational part of the program that calculates mathematical model separately from the user interface

REFERENCES

1. Sanyal S., Chandra S., Kumar S., Roy G. G. An Improved Model of Cored Wire Injection in Steel Melts // ISIJ International, vol. 44. – 2004. – № 7. – P. 1157–1166.
2. Ryabov A.V., Nevolin D.V., Potapov V.I. Mathematical modelling of melt processing with cored wire // Bulletin of the South Ural University, Series “Metallurgy”, vol 13. – 2013. – № 1, – P. 194-198 (in russian).

3. Krasnikov K.S., Samokhvalov S.E., Piptyuk V.P. The computer model of three-dimensional motion of the steel wire in the melt on the ladle-furnace // Mathematical modeling. – 2013. – № 2(29). – P. 95-98 (in ukrainian).
4. Krasnikov K.S., Samokhvalov S.E., Piptyuk V.P., Andrievskyi G.A. The mathematical model of asymmetrical melting of easily fusible supplement // Mathematical modeling. – 2014. – № 2(31). – P. 47-50 (in ukrainian).
5. Samokhvalov S.E., Sokol A.M. The mathematical modeling of transfer processes in the converter. Monograph // Dniprodzerzhinsk: DSTU. – 2013. – 99 p (in ukrainian).
6. Tarasyuk L.I., Morneva V.V., Kazachkov E.O. The method of introduction of wire, stripe into iron-carbon melt // Bulletin of Pryazovskyi State Technical University, Series “Technical sciences”. – 2013. – № 26, – P. 57-61 (in russion).
7. Fedorchenko A.M. Theoretical physics. Mechanics // Kyiv: High School Publishing. – 1971. – P. 76-80 (in ukranian).
8. Gantmakher F.R. Analitical mechanics lections. Learning book for high schools // M.:Fizmatlit. – 2005. – 3 ed. – P. 37-57 (in russian).
9. Patankar S. Numerical heat transfer and fluid flow // New York: Hemisphere Publishing Corporation. – 1980. – P. 44.
10. Ogurtsov A.P., Samokhvalov S.E., Nadrygailo T.Z. Methods of splitting in problems of hydrodynamics and thermomass transfer // Dnepropetrovsk: System technologies. – 2003. – 174 p. (in russian).

 ISSN 1562-9945